UNIT-5
PUMPS

Centrifugal pumps

INTRODUCTION :

- The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps.
- The hydraulic energy is in the form of pressure energy
- If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

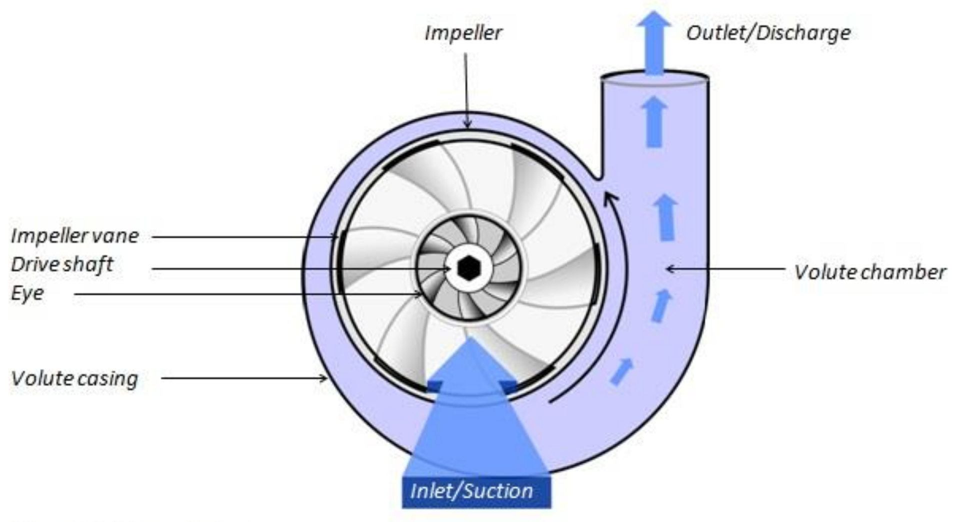
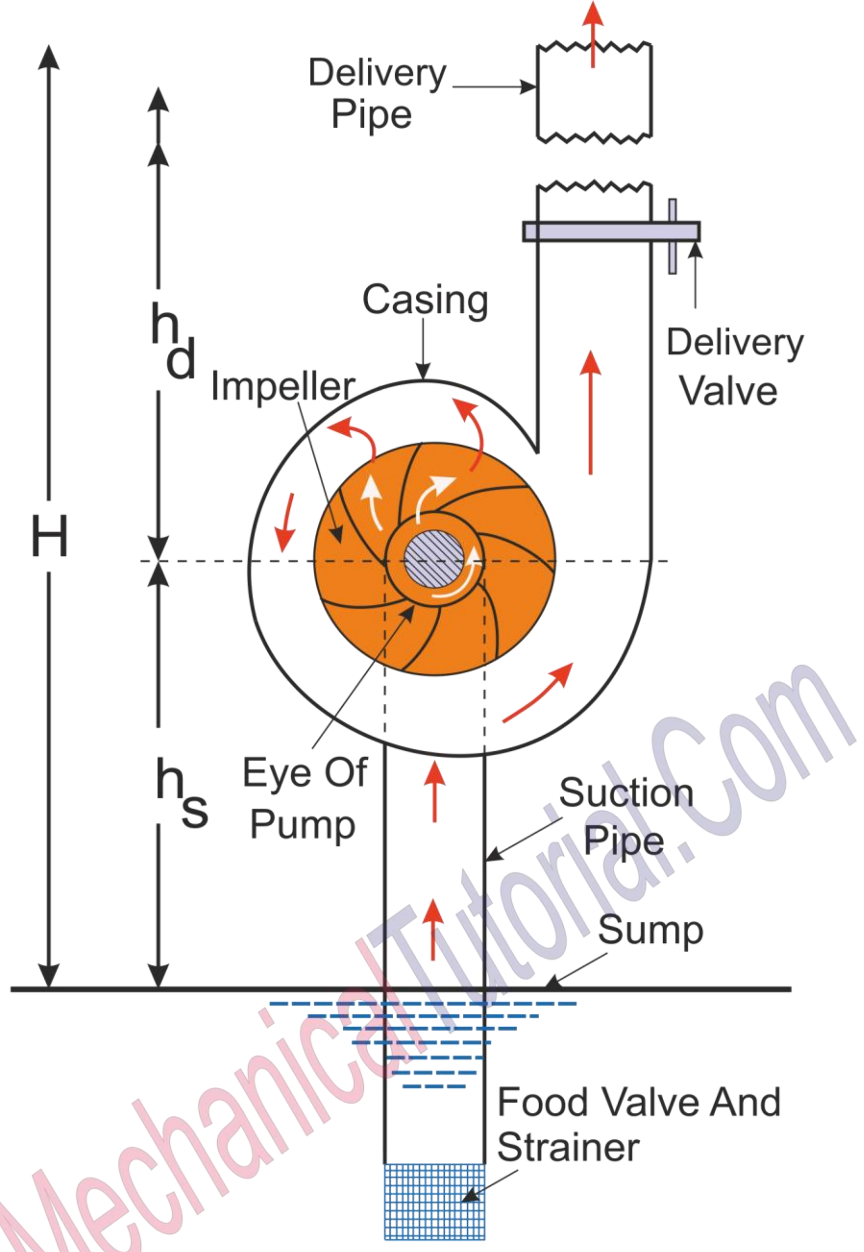


Figure 2. Volute case design

- The centrifugal pump acts as a reverse of an inward radial flow reaction turbine.
- This means that the flow in centrifugal pumps is in the **radial outward directions**

- The centrifugal pump works on the principle of **forced vortex flow** which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place.
- The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point



Line Diagram Of Centrifugal Pump

- Thus at the outlet of the impeller ,where the radius is more, the pressure rise will be more and the liquid will discharge at the outlet with a high pressure head. Due to this high pressure head ,the liquid will lifted to high level.

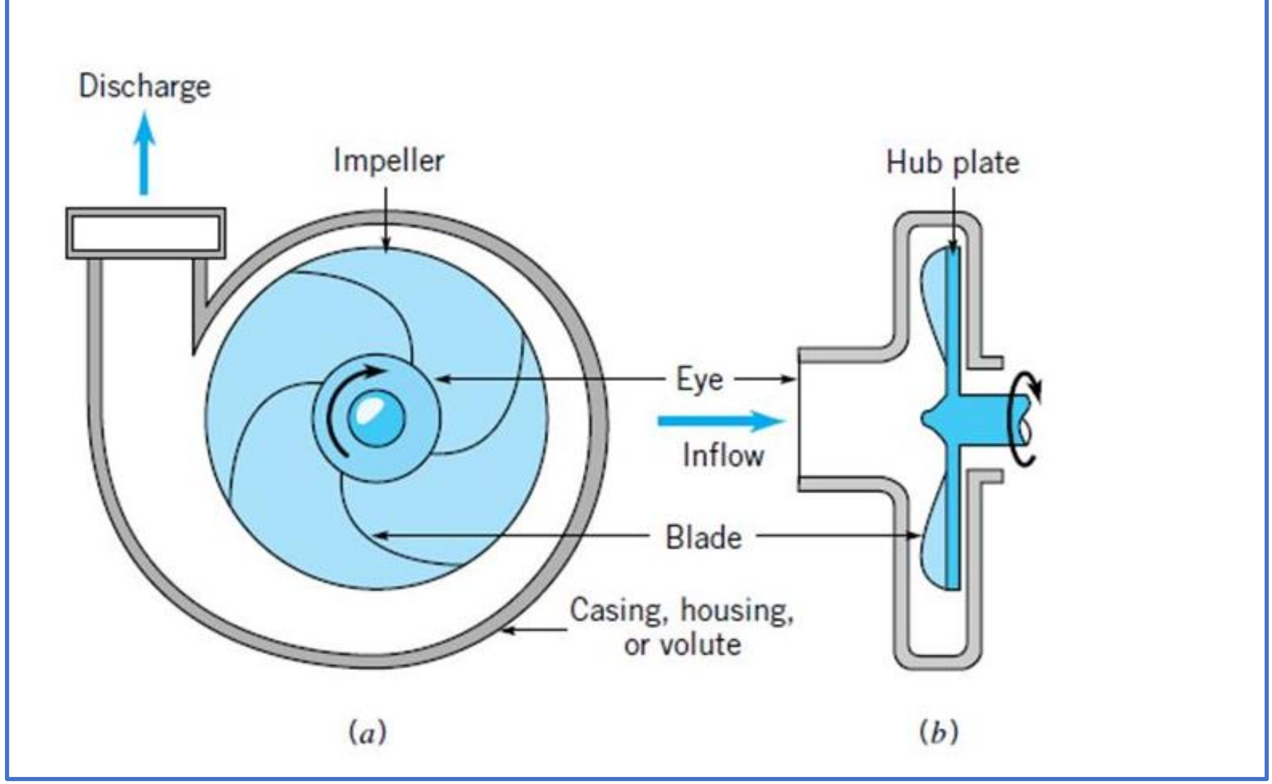
MAIN PARTS OF A CENTRIFUGAL PUMP

The following are the main parts of a centrifugal pump :

1. Impeller.
2. Casing.
3. Suction pipe with a foot valve and a strainer.
4. Delivery pipe.

MAIN PARTS OF A CENTRIFUGAL PUMP

1. Impeller : The rotating part of a centrifugal pump is called 'impeller'. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.



2. Casing. The casing of a centrifugal pump is similar to the casing of a reaction turbine. It is an air-tight passage surrounding the impeller and is designed in such a way that the **kinetic energy of the water discharged** at the outlet of the impeller is converted into **pressure energy** before the water leaves the casing and enters the delivery pipe.

The following three types of the casings are commonly adopted :

(a) Volute Casing : which surrounds the impeller. It is of spiral type in which area of flow increases gradually. The **increase in area** of flow **decreases the velocity** of flow. The **decrease in velocity** increases the **pressure of the water flowing through the casing.**

- It has been observed that in case of volute casing, the efficiency of the pump increases slightly as a large amount of energy is lost due to the **formation of eddies** in this type of casing.

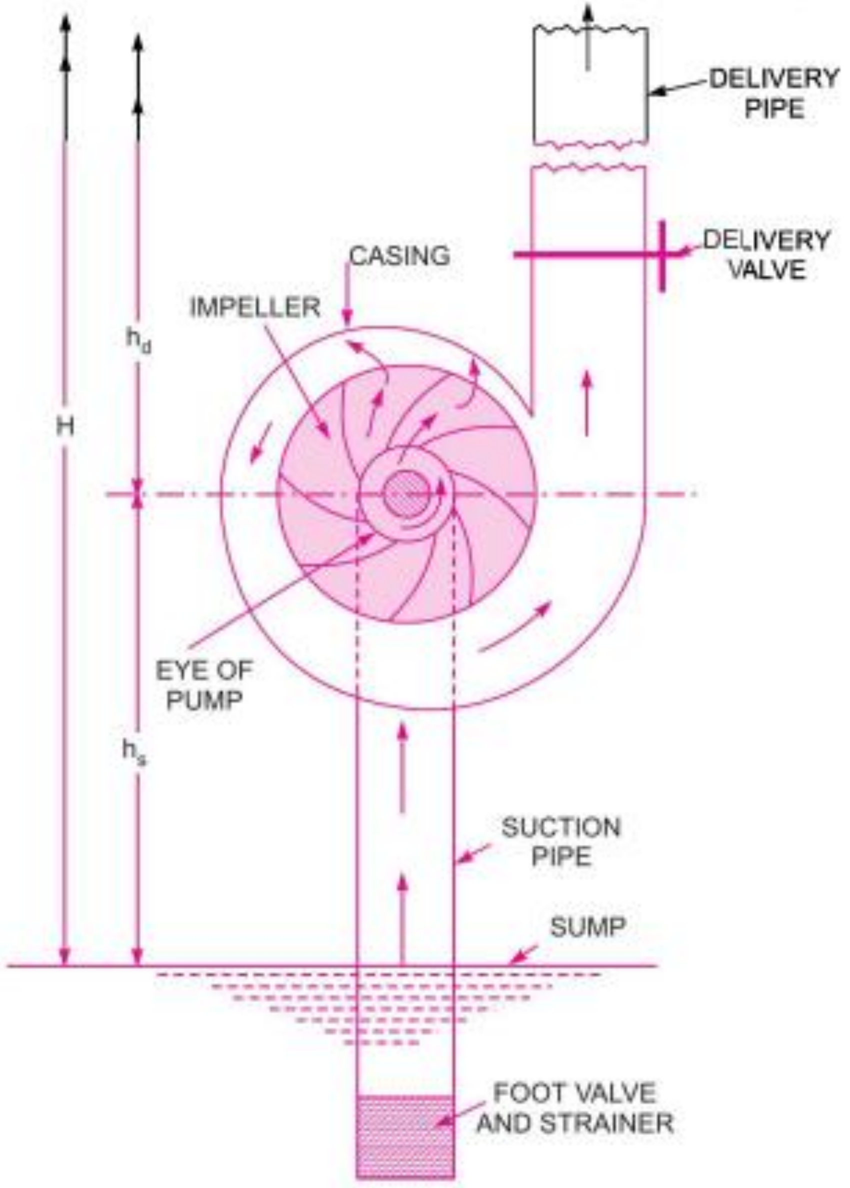
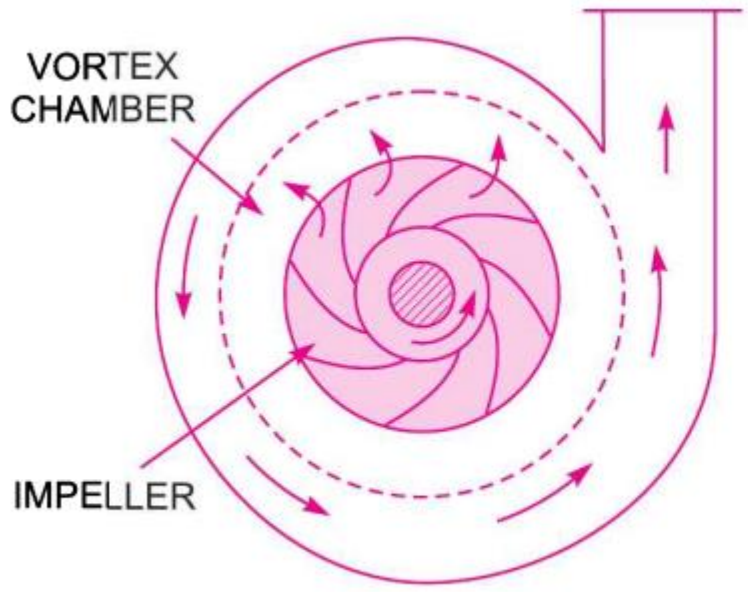


Fig. 19.1 *Main parts of a centrifugal pump.*

(b) Vortex Casing.

If a circular chamber is introduced between the casing and the impeller as shown in Figure the casing is known as Vortex Casing.

- By introducing the circular chamber, the loss of energy due to the **formation of eddies** is reduced to a considerable extent. Thus the efficiency of the pump is more than the efficiency when only volute casing is provided.

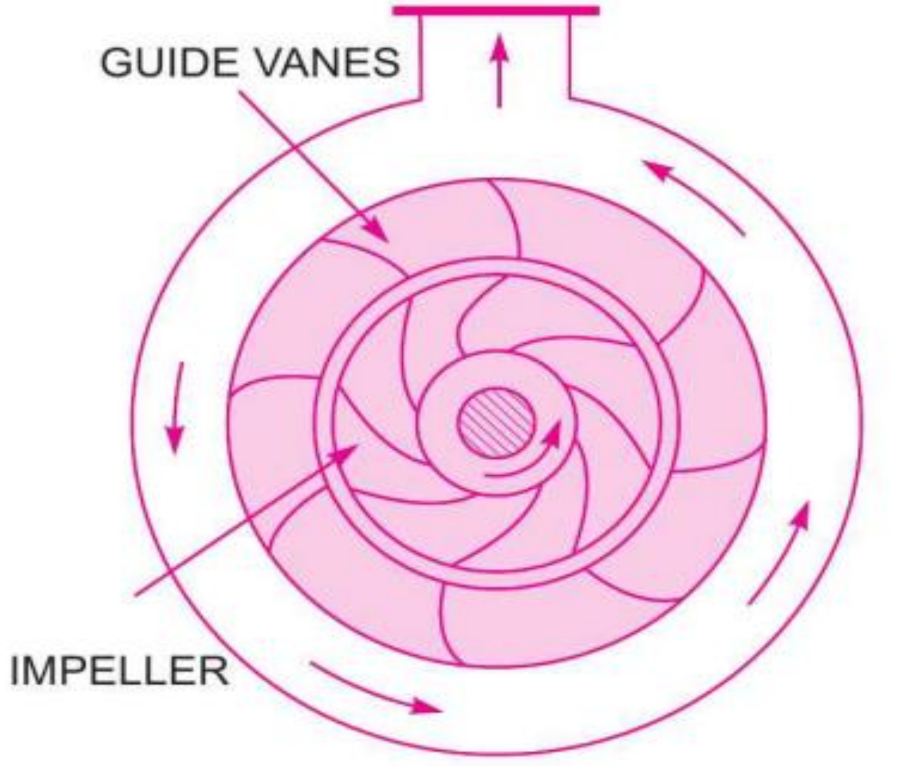


(a) VORTEX CASING

(c) Casing with Guide Blades.

(c) Casing with Guide Blades. This casing is shown in the figure in which the **impeller** is surrounded by a series of guide blades mounted on a ring which is known as **diffuser.**

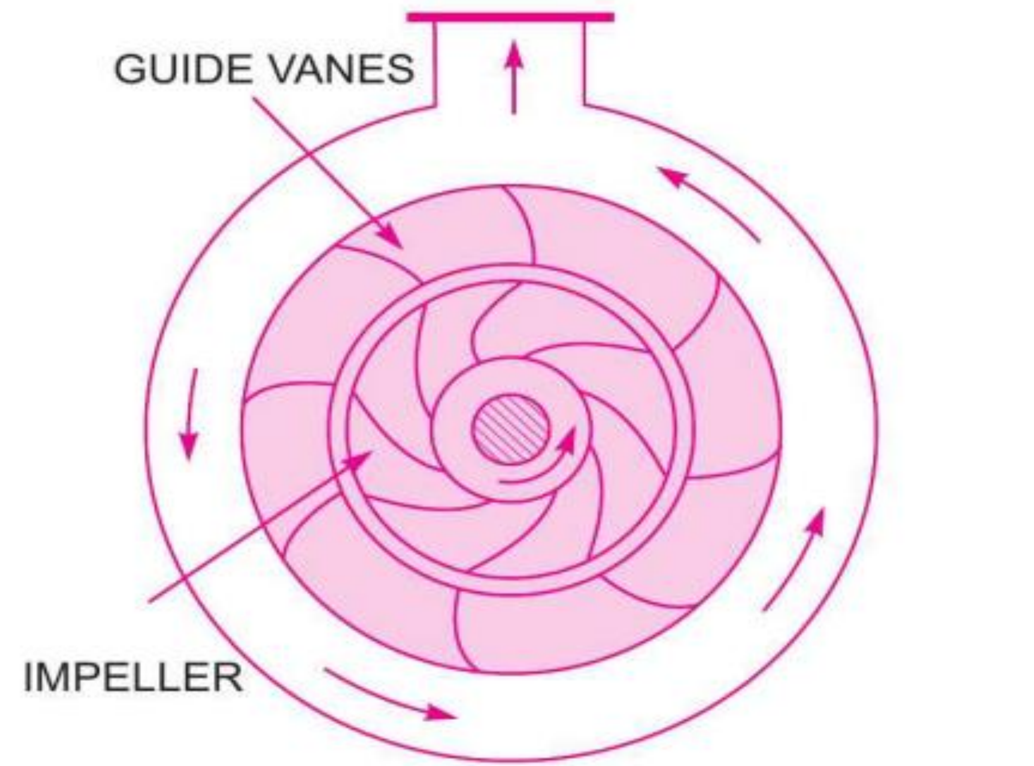
- The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without stock.



(b) CASING WITH GUIDE BLADES

Also the area of the guide vanes increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water.

- The water from the guide vanes then passes through the surrounding casing which is in most of the cases concentric with the impeller as shown in figure



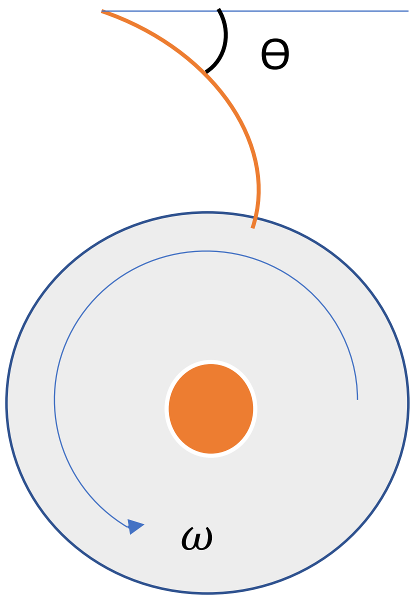
(b) CASING WITH GUIDE BLADES

3. Suction Pipe with a Foot valve and a Strainer.

A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.

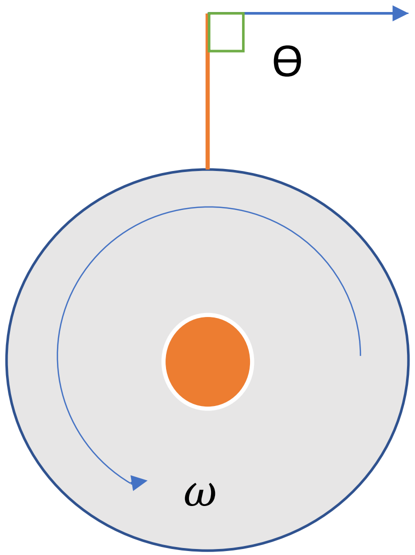
4.Delivery pipe :A Pipe whose one end is connected to the out let of the pump and other end delivers water at the required height is known as delivery pipe

Impeller



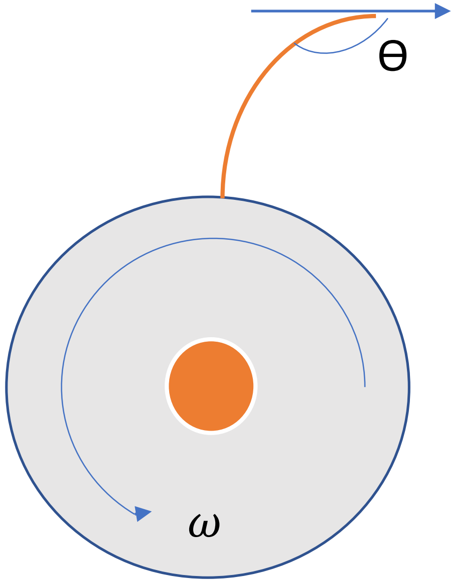
$\theta < 90^0$

Backward vane



$\theta = 90^0$

Radial vane

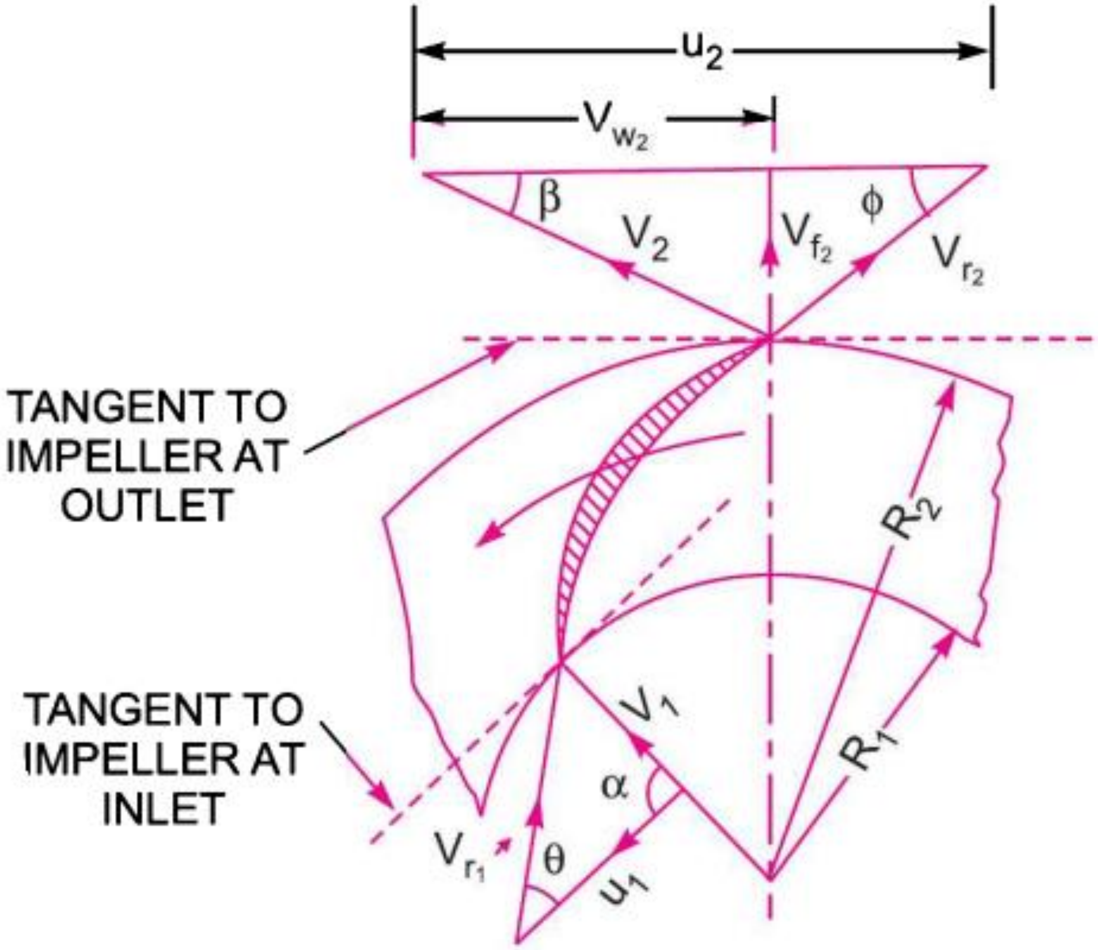


$\theta > 90^0$

Forward vane

Work done by the centrifugal pump or by impeller on water

- In case of the centrifugal pump, work is done by the impeller on the water. The expression for the work done by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as for a turbine.
- The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet.



Velocity triangles at inlet and outlet.

Hence angle **α= 90°** and V_{W1} **=0**

For drawing the velocity triangles, the same notations are used as that for turbines.

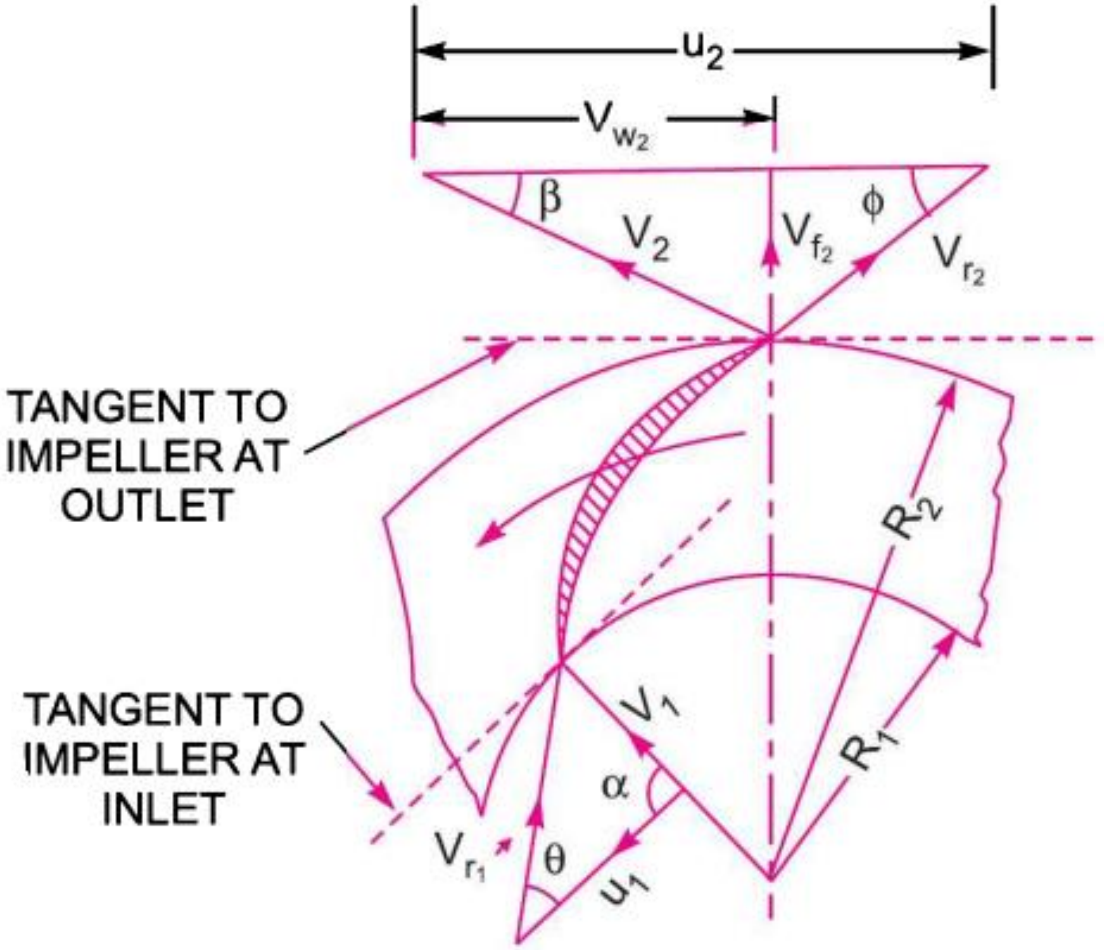
Figure shows the velocity triangles at the inlet and outlet tips of the vanes fixed to an impeller.

Let

N= Speed of the impeller in r.p.m.,

D, = Diameter of impeller at inlet,

U, = Tangential velocity of impeller at inlet,



Velocity triangles at inlet and outlet.

N = Speed of the impeller in r.p.m.,

D_1 = Diameter of impeller at inlet,

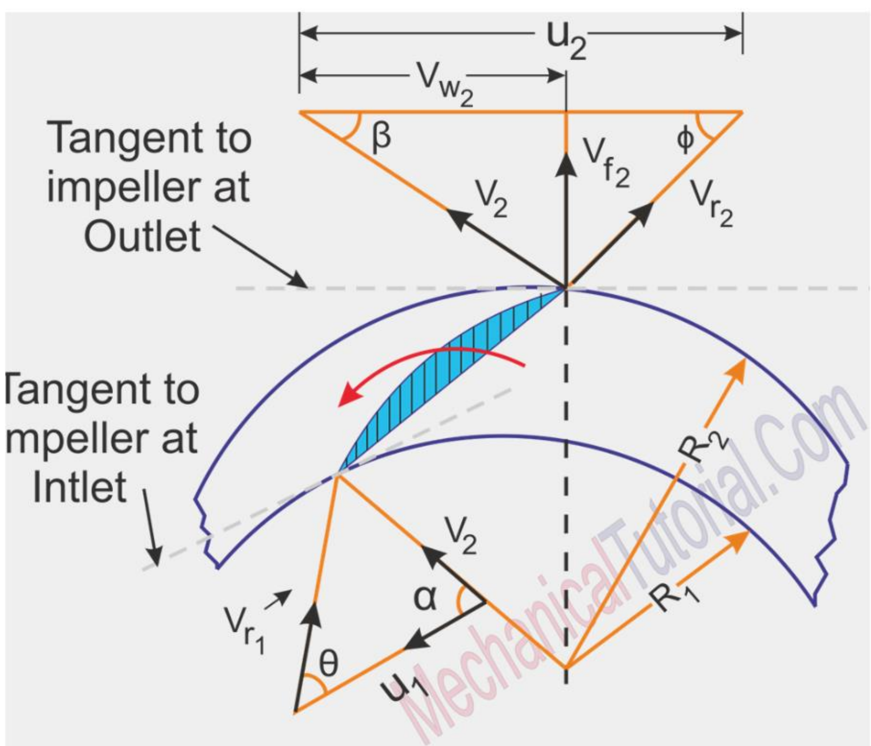
u_1 = Tangential velocity of impeller at inlet, $= \dfrac{\pi D_1 N}{60}$

D_2 = Diameter of impeller at outlet,

u_2 = Tangential velocity of impeller at outlet $= \dfrac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at inlet,

V_{r_1} = Relative velocity of water at inlet,



Velocity triangles at inlet and outlet of Centrifugal Pump

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane,

θ = Angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane, and V_2,

V_{r_2}, β and ϕ are the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha = 90°$ and $V_{w_1} = 0$.

A centrifugal pump is reverse of a radially flow reaction turbine

But in case of a radially inward flow reaction turbine, the work done by the water on the runner per second per unit weight of the water striking per second is given by equation

$$= \frac{1}{g}[V_{w_1}u_1 - V_{w_2}u_2]$$

Work done by the impeller on the water per second per unit weight of water striking per second

= — [Work done in case of turbine]

$$= -\left[\frac{1}{g}\left(V_{w_1}u_1 - V_{w_2}u_2\right)\right] = \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$(\because \quad V_{w_1} = 0 \text{ here}) \qquad = \frac{1}{g}V_{w_2}u_2$$

Work done by impeller on water per second

$$= \frac{W}{g}.V_{w_2}u_2$$

W = Weight of water = $\rho \times g \times Q$

Q = Volume of water

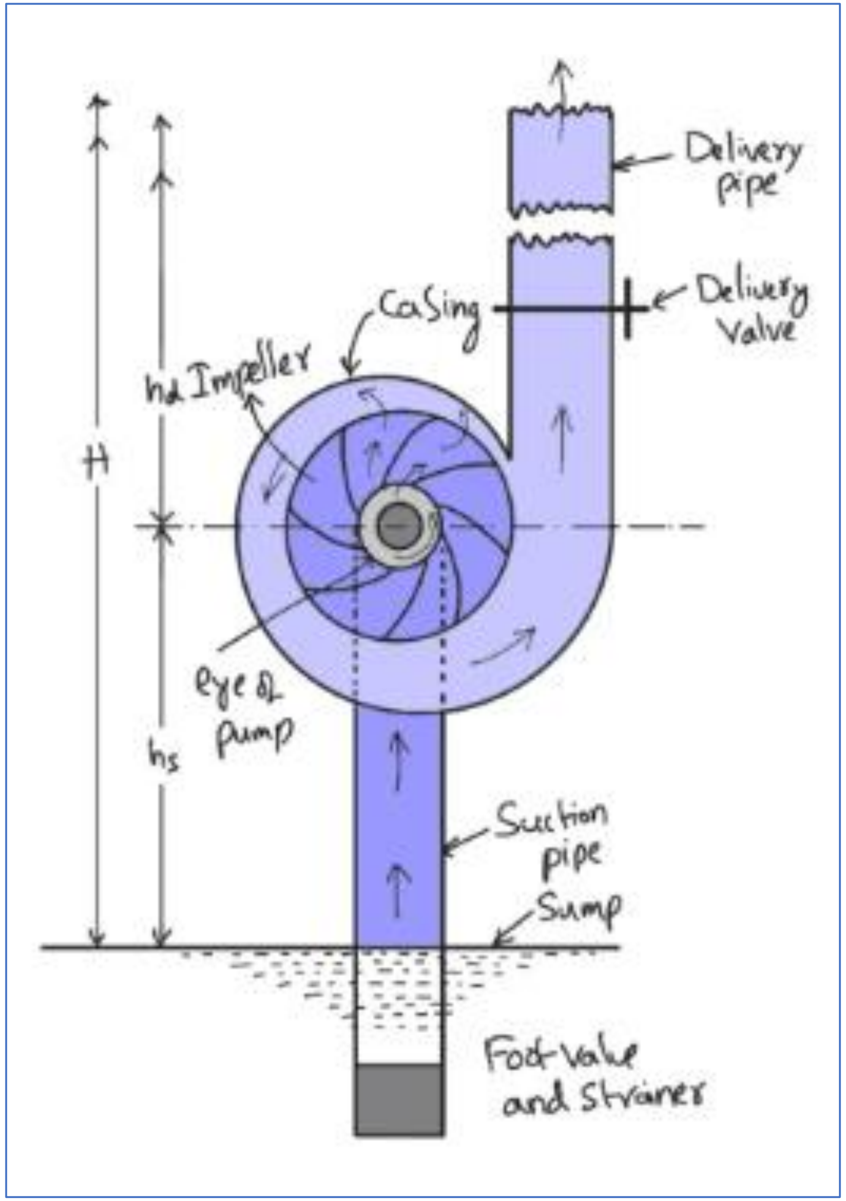
Q = Area × Velocity of flow $= \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}$

where B_1 and B_2 are width of impeller at inlet and outlet and V_{f_1} and V_{f_2} are velocities of flow at inlet and outlet.

DEFINITIONS OF HEADS AND EFFICIENCIES OF A CENTRIFUGAL PUMP

1. Suction Head h_s

It is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted as shown in Figure This height is also called suction lift and is denoted by h_s



2. Delivery Head h_d. The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head.

3. Static Head (Hs). The sum of suction head and delivery head is known as static head.

$$H_S = h_s + h_d$$

4. Manometric Head (H_m). The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions :

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing}$$

$$= \frac{V_{w_2} u_2}{g} \quad \text{...if loss of pump is zero}$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad \text{...}$$

$\frac{p_o}{\rho g}$ = Pressure head at outlet of the pump = h_d

$\frac{V_o^2}{2g}$ = Velocity head at outlet of the pump

= Velocity head in delivery pipe = $\frac{V_d^2}{2g}$

Z_o = Vertical height of the outlet of the pump from datum line, and

$\frac{p_i}{\rho g}, \frac{V_i^2}{2g}$, Z_i = Corresponding values of pressure head, velocity head and datum head at the inlet of the pump,

$h_s, \frac{V_s^2}{2g}$ and Z_s respectively.

(c) $$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$$

h_s = Suction head, h_d = Delivery head,
h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe, and
V_d = Velocity of water in delivery pipe.

5. Efficiencies of a Centrifugal Pump.

- In case of a centrifugal pump, the power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller.
- From the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to the water.

The following are the important efficiencies of a centrifugal pump :

(a) Manometric efficiency,
(b) Mechanical efficiency, and
(c) Overall efficiency,

(*a*) **Manometric Efficiency (η_{man}).** The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency. Mathematically, it is written as

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2}$$

$$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

$$\text{The power given to water at outlet of the pump} = \frac{WH_m}{1000} \text{ kW}$$

The power at the impeller $= \dfrac{\text{Work done by impeller per second}}{1000}\text{kW}$

$$= \frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}\text{kW}$$

$$\eta_{man} = \frac{\dfrac{W \times H_m}{1000}}{\dfrac{W}{g} \times \dfrac{V_{w_2} \times u_2}{1000}} = \frac{g \times H_m}{V_{w_2} \times u_2}.$$

(*b*) **Mechanical Efficiency (η_m).** The power at the shaft of the centrifugal pump is more than the power available at the impeller of the pump. The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency. It is written as

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

$$\text{The power at the impeller in kW} = \frac{\text{Work done by impeller per second}}{1000}$$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}}$$

(*c*) **Overall Efficiency (η_0).** It is defined as ratio of power output of the pump to the power input to the pump. The power output of the pump in kW

$$= \frac{\text{Weight of water lifted} \times H_m}{1000} = \frac{WH_m}{1000}$$

Power input to the pump = Power supplied by the electric motor

= S.P. of the pump.

$$\eta_o = \frac{\left(\dfrac{WH_m}{1000}\right)}{\text{S.P.}}$$

$$\eta_o = \eta_{man} \times \eta_m.$$

Problem 19.1 *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Solution. Given :

Internal diameter of impeller, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

External diameter of impeller, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

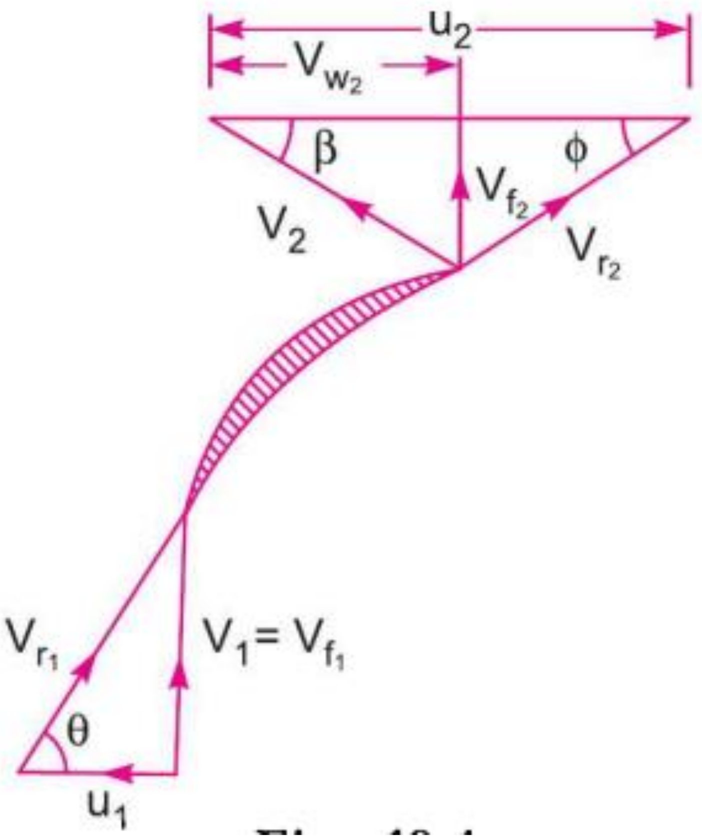
Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$



Tangential velocity of impeller at inlet and outlet are,

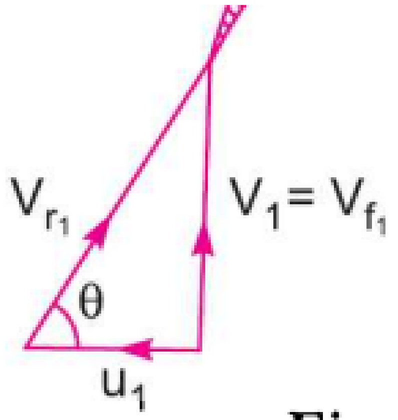
$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

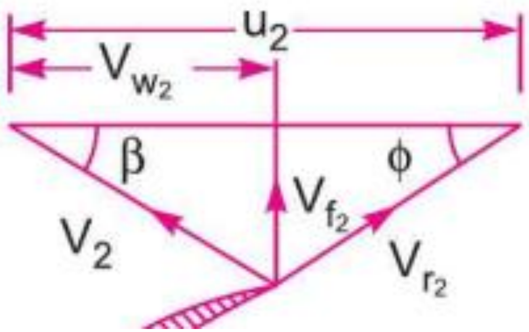
From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore$ $V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore$ $V_{f_2} = V_{f_1} = 4.57$ m/s.



From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2 - V_{w_2}} = \dfrac{4.57}{25.13 - V_{w_2}}$



$$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30^\circ} = 7.915$$

$$\therefore \quad V_{w_2} = 25.13 - 7.915 = 17.215 \text{ m/s.}$$

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \mathbf{44.1\ Nm/N.\ Ans.}$$

MINIMUM SPEED FOR STARTING A CENTRIFUGAL PUMP

- If the pressure rise in the impeller is more than or equal to manometric head (**H_{man}**), the centrifugal pump will start delivering water. Otherwise, the pump will not discharge any water, though the impeller is rotating.
- When impeller is rotating, the water in contact with the impeller is also rotating. This is the case of **forced vortex.**
- In case of forced vortex, the centrifugal head or head due to pressure rise in the impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

where ωr_2 = Tangential velocity of impeller at outlet = u_2, and
ωr_1 = Tangential velocity of impeller at inlet = u_1.

$\therefore$ Head due to pressure rise in impeller $= \dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g}$

The flow of water will commence only if

Head due to pressure rise in impeller $\geq H_m$ or $\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} \geq H_m.$ → **(a)**

For minimum speed, we must have $\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} = H_m$

Manometric efficiency : $\eta_{man} = \dfrac{gH_m}{V_{w_2} u_2}$

$$H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}. \rightarrow \textbf{(b)}$$

Substitute (b) in (a)

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

$$u_2 = \frac{\pi D_2 N}{60} \text{ and } u_1 = \frac{\pi D_1 N}{60}.$$

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

$$u_2 = \frac{\pi D_2 N}{60} \text{ and } u_1 = \frac{\pi D_1 N}{60}.$$

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \times \pi D_2 N}{g \times 60}$$

Dividing by $\frac{\pi N}{g \times 60}$

we get $$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{man} \times V_{w_2} \times D_2$$

$$\frac{\pi N}{120}[D_2^2 - D_1^2] = \eta_{man} \times V_{w_2} \times D_2$$

$$N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}$$

minimum starting speed of the centrifugal pump.

Problem 19.13 *The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. Determine the minimum starting speed of the pump if it works against a head of 30 m.*

Dia. of impeller at inlet, $D_1 = 30 \text{ cm} = 0.30 \text{ m}$

Dia. of impeller at outlet, $D_2 = 60 \text{ cm} = 0.60 \text{ m}$

Head, $H_m = 30 \text{ cm}$

Let the minimum starting speed $= N$

$$u_2 = \frac{\pi \times D_2 \times N}{60} = \frac{\pi \times 0.6 \times N}{60} = 0.03141\, N$$

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

$$u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times .3 \times N}{60} = 0.0157\, N$$

$$\frac{1}{2g}(0.3141\, N)^2 - \frac{1}{2g}(.0157\, N)^2 = 30$$

$$(.03141\, N)^2 - (.0157\, N)^2 = 30 \times 2 \times g = 30 \times 2 \times 9.81$$

$$N^2 = \frac{30 \times 2 \times 9.81}{\left(.03141^2 - .0157^2\right)} \qquad N = \sqrt{795297.9} = \mathbf{891.8 \text{ r.p.m. Ans.}}$$

Problem 19.14 *The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. The velocity of flow at outlet is 2.0 m/s and the vanes are set back at an angle of 45° at the outlet. Determine the minimum starting speed of the pump if the manometric efficiency is 70%.*

Diameter at inlet, $D_1 = 30 \text{ cm} = 0.30 \text{ m}$

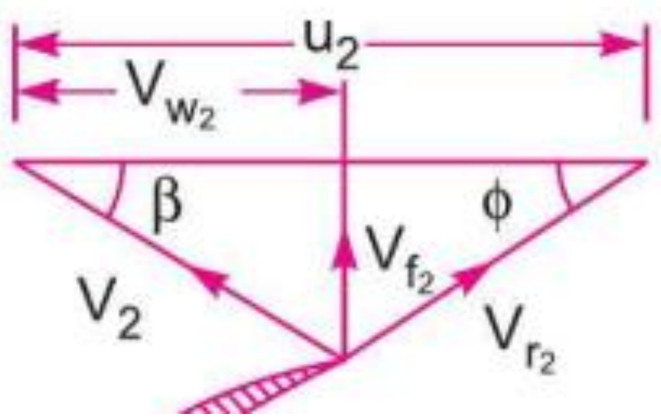
Diameter at outlet, $D_2 = 60 \text{ cm} = 0.60 \text{ m}$

Velocity of flow at outlet, $V_{f_2} = 2.0 \text{ m/s}$

Vane angle at outlet, $\phi = 45°$

Manometric efficiency, $\eta_{man} = 70\% = 0.70.$

Let the minimum starting speed $= N$.



$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \qquad u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = \frac{2.0}{\tan 45°} = 2.0$$

$$V_{w_2} = u_2 - 2.0$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.60 \times N}{60} = 0.03141 \text{ N} \qquad V_{w_2} = (0.03141N - 2.0).$$

$$N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}$$

minimum starting speed of the centrifugal pump.

$$N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]} = \frac{120 \times 0.70 \times (.03141\,N - 2.0) \times 0.6}{\pi\left[.6^2 - .3^2\right]}$$

$$= \frac{50.4(.03141\,N - 2.0)}{\pi[.36 - .09]} = 59.417\ [.03141\ \text{N} - 2.0]$$

$$= 1.866\ \text{N} - 118.834$$

$$1.866\,N - N = 118.834 \text{ or } .886\,N = 118.834$$

$$N = \frac{118.834}{0.866} = \mathbf{137.22\ r.p.m.\ Ans.}$$

MULTISTAGE CENTRIFUGAL PUMPS

If a centrifugal pump consists of two or more impellers, the pump is called a multistage centrifugal pump.

The impellers may be mounted on the same shaft or on different shafts.

A multistage pump is having the following two important functions :

1. To produce a high head

2. To discharge a large quantity of liquid.

- If a **high head is to be developed**, the impellers are connected in series (or on the same shaft)
- While for **discharging large quantity of liquid**, the impellers (or pumps) are connected in parallel.

Multistage Centrifugal Pumps for High Heads.

For developing a high head, a number of impellers are mounted in series or on the same shaft as shown in Figure

The water from suction pipe enters the **1st impeller at inlet** and is discharged at outlet with increased pressure. The water with increased pressure from the outlet of the **1st impeller** is taken to the **inlet of the 2nd impeller** with the help of a connecting pipe as shown in Figure

At the outlet of the 2nd impeller, the pressure of water will be more than the pressure of water at the outlet of the 1st impeller. Thus if more impellers are mounted on the **same shaft**, the pressure at the outlet will be increased further.

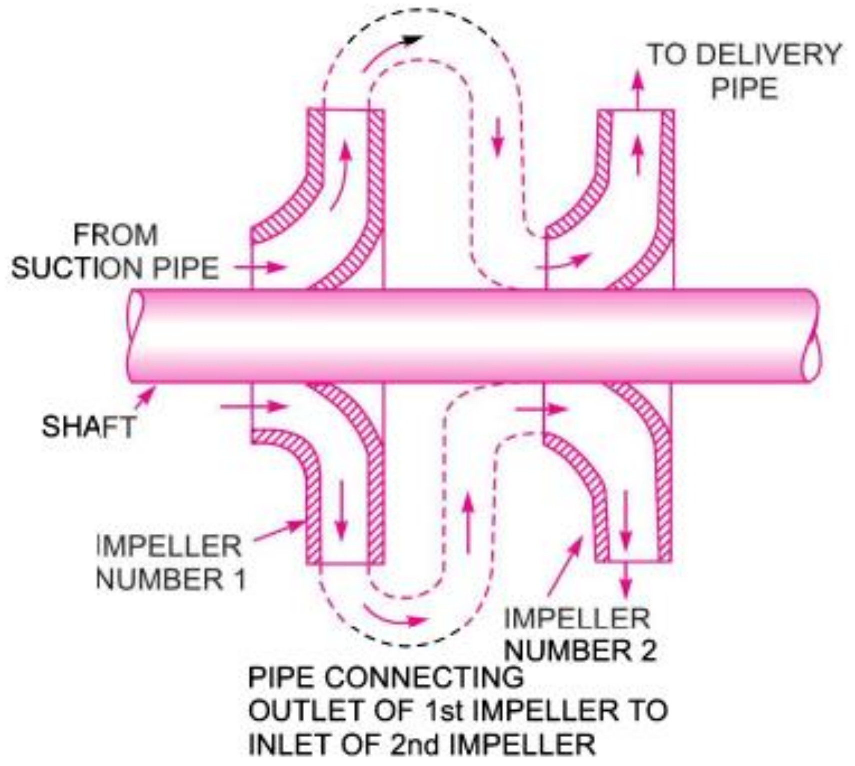


Fig. 19.12 *Two-stage pumps with impellers in series.*

Let n = Number of identical impellers mounted on the same shaft.

H_m = Head developed by each impeller.

Then total head developed

$$= n \times H_m$$

The discharge passing through each impeller is same

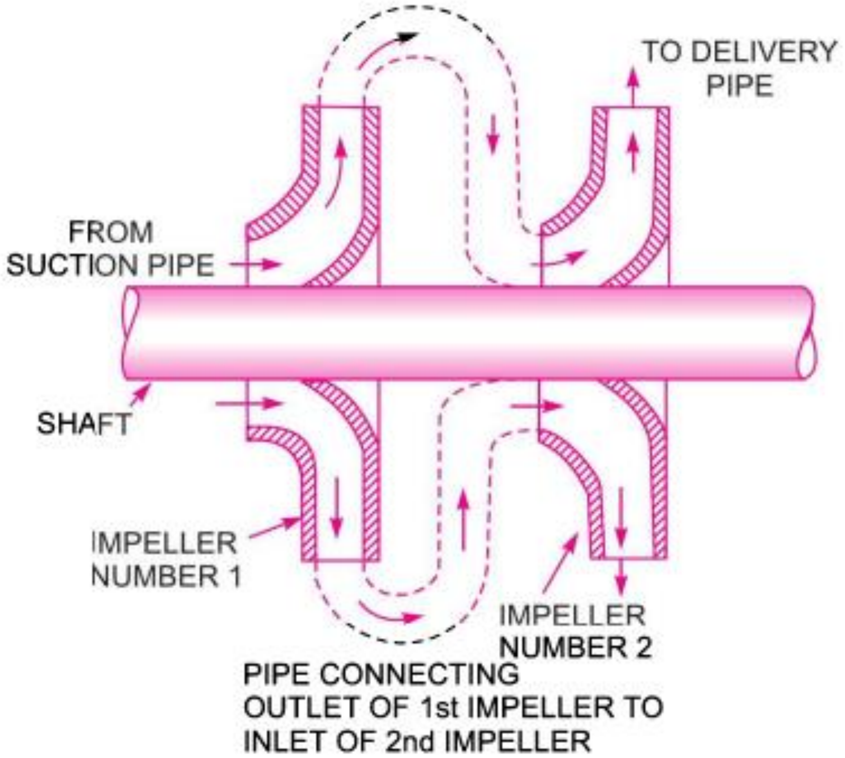


Fig. 19.12 *Two-stage pumps with impellers in series.*

19.6.2 Multistage Centrifugal Pumps for High Discharge. For obtaining high discharge, the pumps should be connected in parallel as shown in Fig. 19.13. Each of the pumps lifts the water from a common pump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each of the pump is working against the same head.

Let n = Number of identical pumps arranged in parallel.

Q = Discharge from one pump.

∴ Total discharge $= n \times Q$

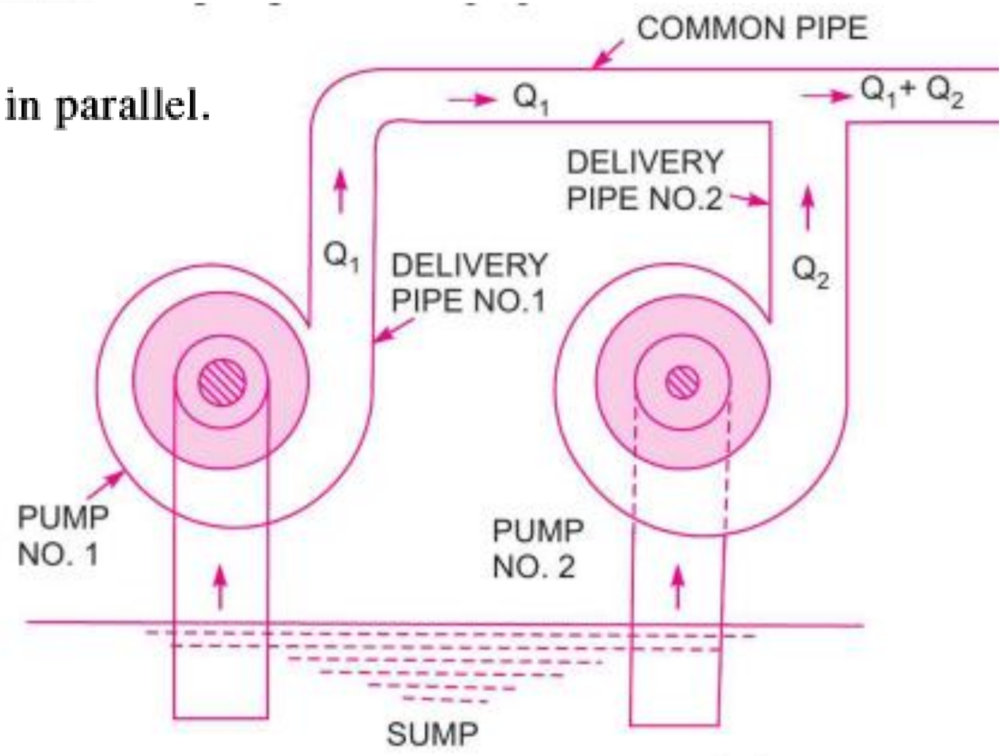


Fig. 19.13 *Pumps in parallel.*

SPECIFIC SPEED OF A CENTRIFUGAL PUMP (N_s)

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver *one cubic metre* of liquid per second against a head of *one metre*. It is denoted by 'N_s'.

Expression for Specific Speed for a Pump.

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= \pi D \times B \times V_f \text{ or } Q \propto D \times B \times V_f$$

where D = Diameter of the impeller of the pump and
B = Width of the impeller.

We know that $B \propto D$

$\therefore$ From equation (*i*), we have $Q \propto D^2 \times V_f$

We also know that tangential velocity is given by

$$u = \frac{\pi DN}{60} \propto DN$$

Now the tangential velocity (u) and velocity of flow (V_f) are related to the manometric head (H_m) as

$$u \propto V_f \propto \sqrt{H_m}$$

$$u = \frac{\pi D N}{60} \propto DN$$

$$\sqrt{H_m} \propto DN \text{ or } D \propto \frac{\sqrt{H_m}}{N}$$

we have $Q \propto D^2 \times V_f$

$$Q \propto \frac{H_m}{N^2} \times V_f$$

$$\propto \frac{H_m}{N^2} \times \sqrt{H_m} \qquad [\because \text{ From equation } (iv),\ V_f \propto \sqrt{H_m}\ ]$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \frac{H_m^{3/2}}{N^2}$$

where K is a constant of proportionality.

If $H_m = 1$ m and $Q = 1$ m^3/s, N becomes $= N_s$.

$$1 = K \frac{1^{3/2}}{N_s^2} = \frac{K}{N_s^2}$$

$$K = N_s^2$$

Substituting the value of K in equation (v), we get

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2} \quad \text{or} \quad N_s^2 = \frac{N^2 Q}{H_m^{3/2}}$$

$$\therefore \qquad N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

Before manufacturing the large sized pumps, their models which are in complete similarity with the actual pumps (also called prototypes) are made. Tests are conducted on the models and performance of the prototypes are predicted. The complete similarity between the model and actual pump (prototype) will exist if the following conditions are satisfied :

1. Specific speed of model = Specific speed of prototype

$$(N_s)_m = (N_s)_p \quad \text{or} \quad \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_p$$

2. Tangential velocity (u) is given by $u = \dfrac{\pi DN}{60}$ also $u \propto \sqrt{H_m}$

$$\sqrt{H_m} \propto DN$$

$$\frac{\sqrt{H_m}}{DN} = \text{Constant}$$

$$\left(\frac{\sqrt{H_m}}{DN}\right)_m = \left(\frac{\sqrt{H_m}}{DN}\right)_p$$

$$Q \propto D^2 \times V_f$$
$$\propto D^2 \times D \times N$$
$$\propto D^3 \times N$$

$$\frac{Q}{D^3 N} = \text{Constant} \text{ or } \left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p$$

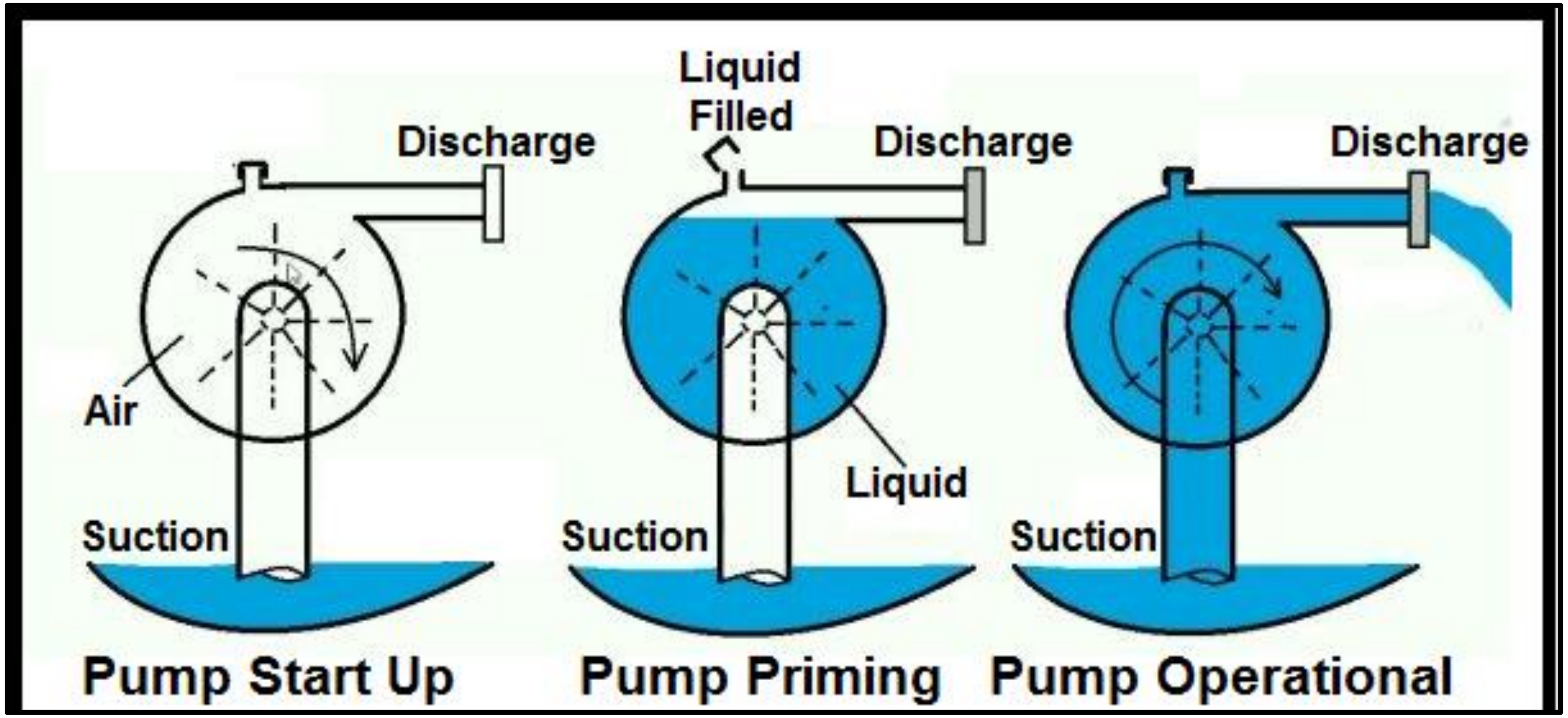
4. Power of the pump, $P = \dfrac{\rho \times g \times Q \times H_m}{75}$

$$P \propto Q \times H_m$$
$$\propto D^3 \times N \times H_m \qquad (\because \quad Q \propto D^3 N)$$
$$\propto D^3 N \times D^2 N^2 \qquad (\because \quad \sqrt{H_m} \propto DN)$$
$$\propto D^5 N^3$$

$$\frac{P}{D^5 N^3} = \text{Constant} \quad \text{or} \quad \left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p$$

PRIMING OF A CENTRIFUGAL PUMP

- Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe up to the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump.
- Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.



- The work done by the impeller per unit weight of liquid per sec is known as the head generated by the pump
- Head generated by the pump given by $= \frac{1}{g} V_{w_2} u_2$ metre.
- This equation is independent of the density of the liquid. This means that **when pump** is running in air, the head generated is in terms of metre of air.
- If the pump is primed with water, the head generated is same metre of water. But as the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible and hence the water may not be sucked from the pump.

To avoid this difficulty, priming is necessary.

CHARACTERISTIC CURVES OF CENTRIFUGAL PUMPS

- Characteristic curves of centrifugal pumps are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump.
- These curves are necessary to predict the behavior and performance of the pump when the pump is working under different **flow rate**, **head and speed**. The following are the important characteristic curves for pumps :

The following are the important characteristic curves for pumps :

1. Main characteristic curves,

2. Operating characteristic curves, and

3. Constant efficiency or Muschel curves.

1) Main Characteristic Curves.

The main characteristic curves of a centrifugal pump consists of variation of head **(manometric head, H)**, **power** and **discharge with respect to speed.**

- For plotting curves of manometric head versus speed, discharge is kept constant.
- For plotting curves of discharge versus speed, manometric head (H) is kept constant. And
- for plotting curves of power versus speed, the manometric head and discharge are kept constant. Figure shows main characteristic curves of a pump.

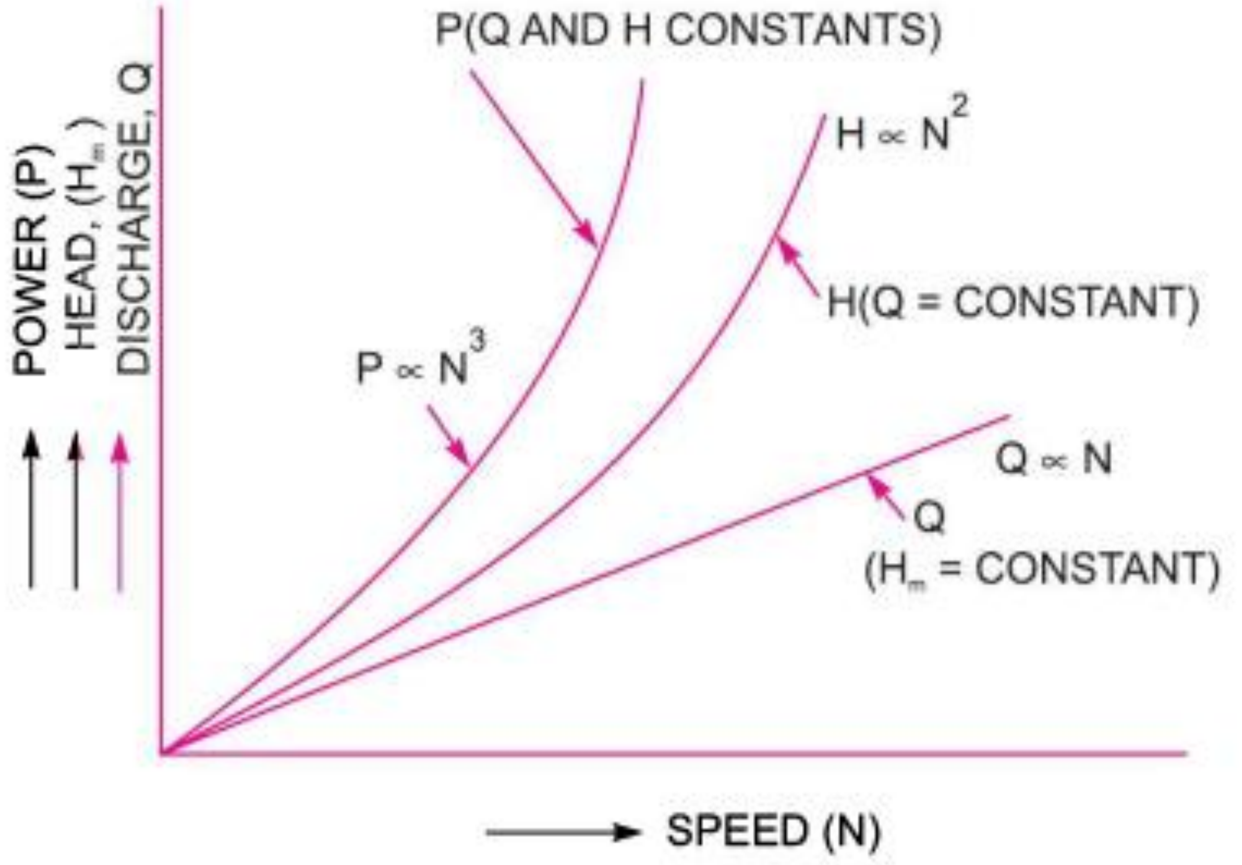


Fig. 19.14 *Main characteristic curves of a pump.*

For plotting the graph of H_m versus speed (N), the discharge is kept constant.

2. Tangential velocity (u) is given by $u = \frac{\pi DN}{60}$ also $u \propto \sqrt{H_m}$

$$\therefore \quad \sqrt{H_m} \propto DN$$

$$\therefore \quad \frac{\sqrt{H_m}}{DN} = \text{Constant} \quad \text{Or}$$

$$H_m \propto N^2.$$

This means that head developed by the pump is proportional to N^2

Hence the curve of H_m v/s N is a parabolic curves

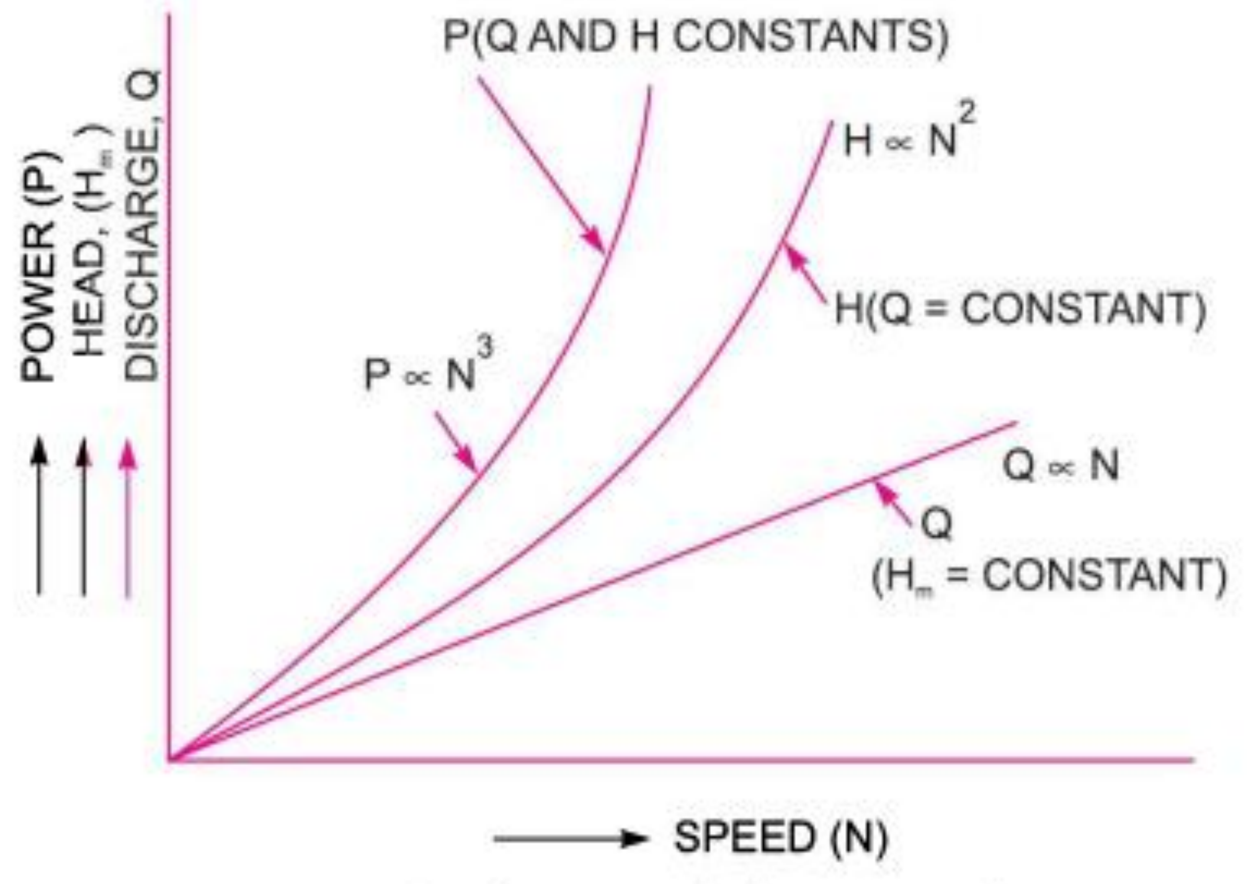


Fig. 19.14 *Main characteristic curves of a pump.*

$$P = \frac{\rho \times g \times Q \times H_m}{75}$$

$$P \propto Q \times H_m$$

$$\propto D^3 \times N \times H_m \qquad (\because \quad Q \propto D^3N)$$

$$\propto D^3N \times D^2N^2 \qquad (\because \quad \sqrt{H_m} \propto DN)$$

$$\propto D^5N^3$$

$$\frac{P}{D^5N^3} = \text{Constant} \quad \text{or} \quad \left(\frac{P}{D^5N^3}\right)_m = \left(\frac{P}{D^5N^3}\right)_p \qquad \text{...(19.22)}$$

Hence $P \propto N^3$. This means that the curve

P v/s N is a cubic curve as shown i

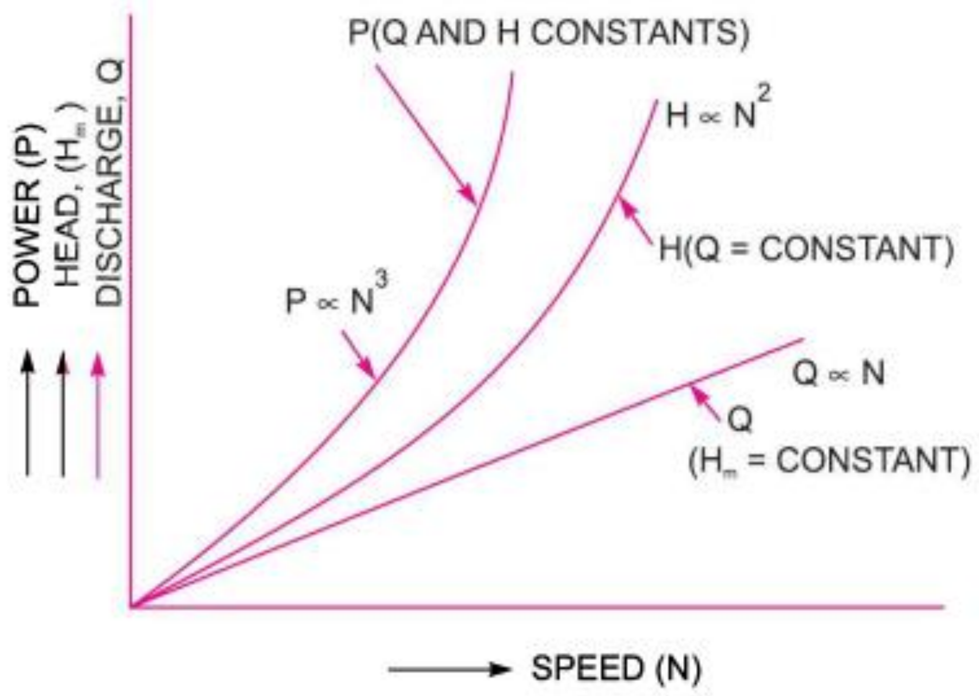


Fig. 19.14 *Main characteristic curves of a pump.*

. From equation (*ii*) of Art. 19.7.1, we have

$$Q \propto D^2 \times V_f$$
$$\propto D^2 \times D \times N$$
$$\propto D^3 \times N$$

. $$\frac{Q}{D^3 N} = \text{Constant} \quad \text{or} \quad \left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p$$

, **shows that** $\frac{Q}{D^3 N}$ **= constant.**

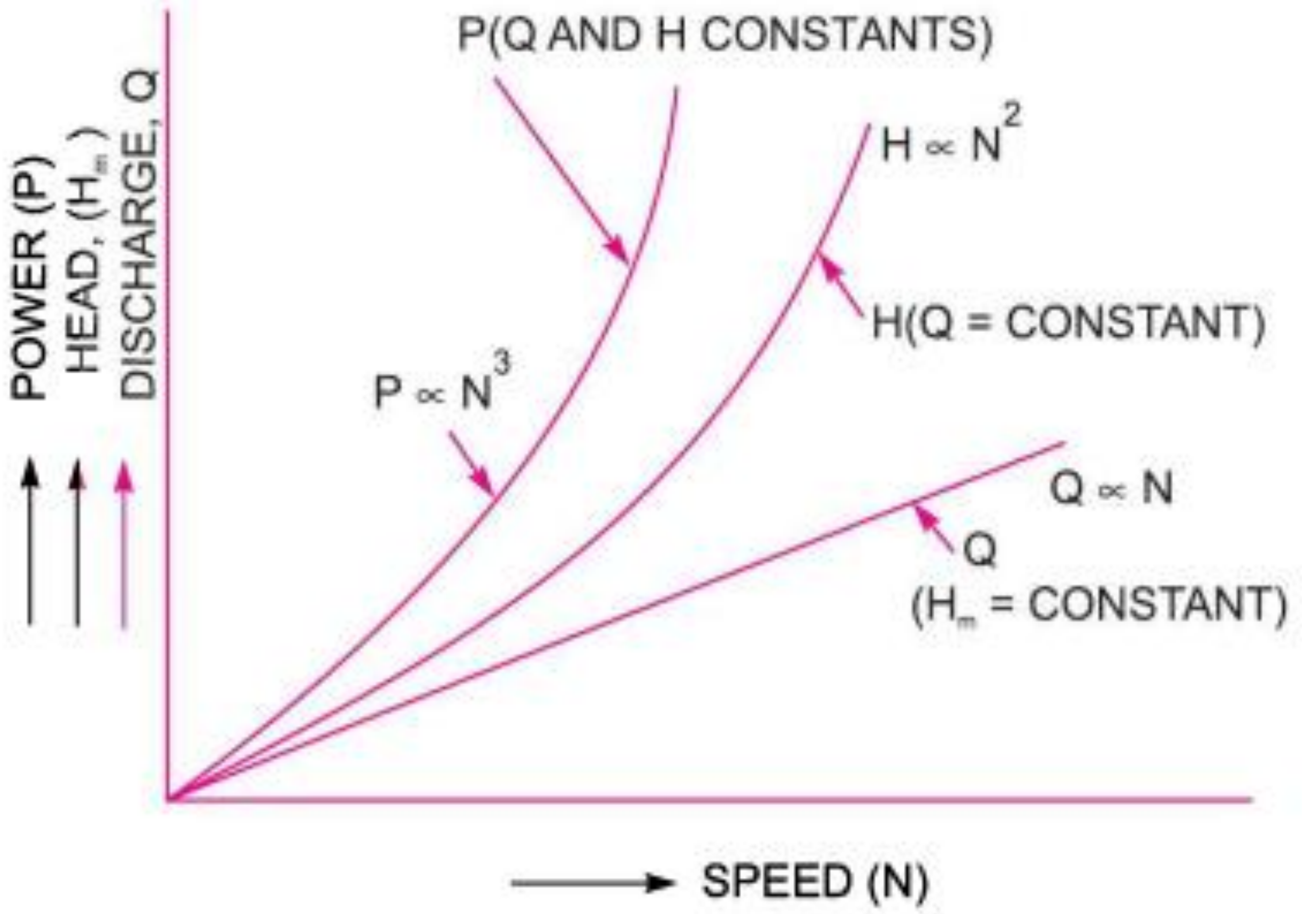


Fig. 19.14 *Main characteristic curves of a pump.*

2. Operating Characteristic Curves.

- If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump.
- The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.

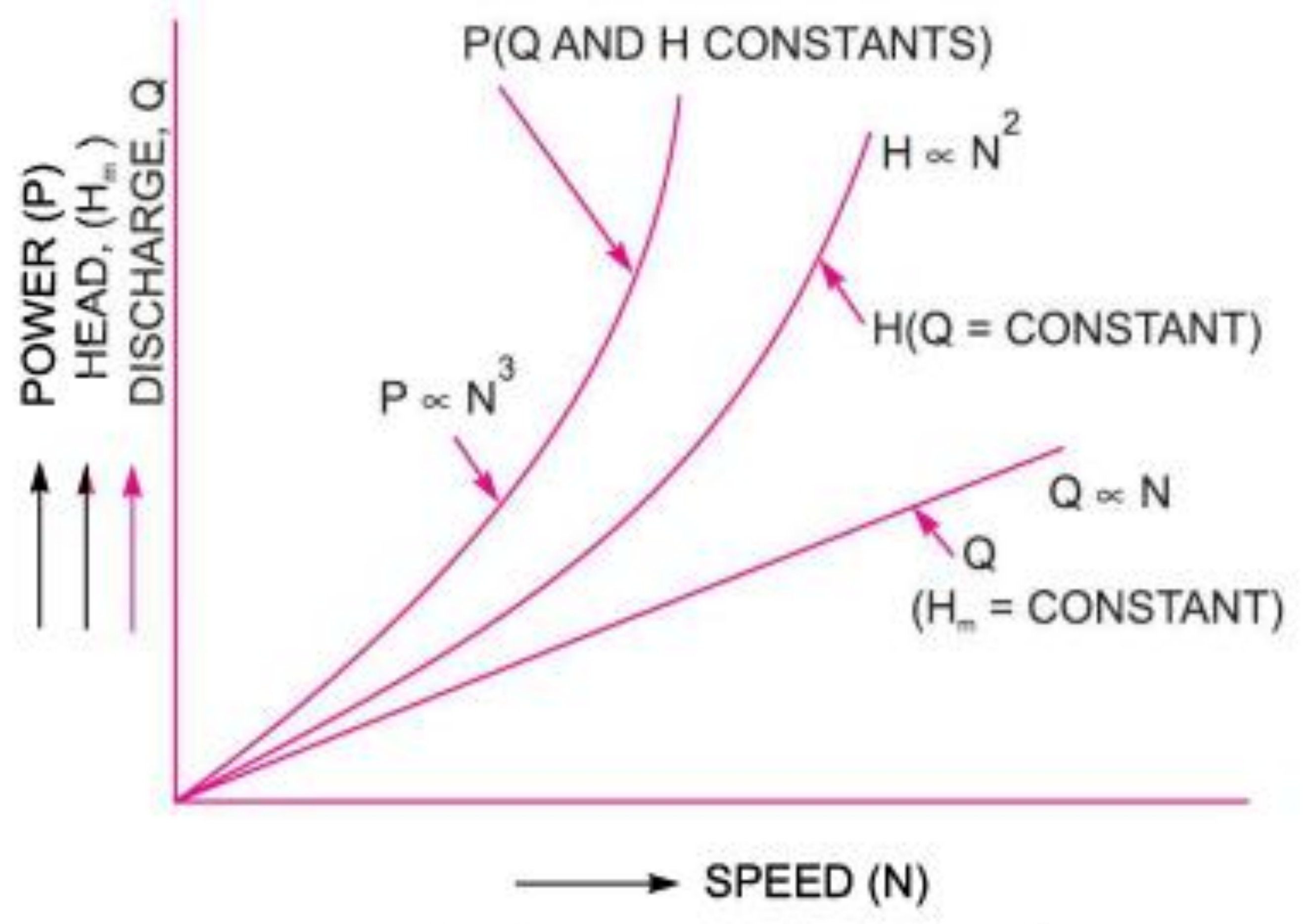


Fig. 19.14 *Main characteristic curves of a pump.*

- The head curve will have maximum value of head when discharge is zero.
- The output power curve will start from origin as at Q = 0, output power (pQgH) will be zero
- The efficiency curve will start from origin as at Q = 0, η = 0

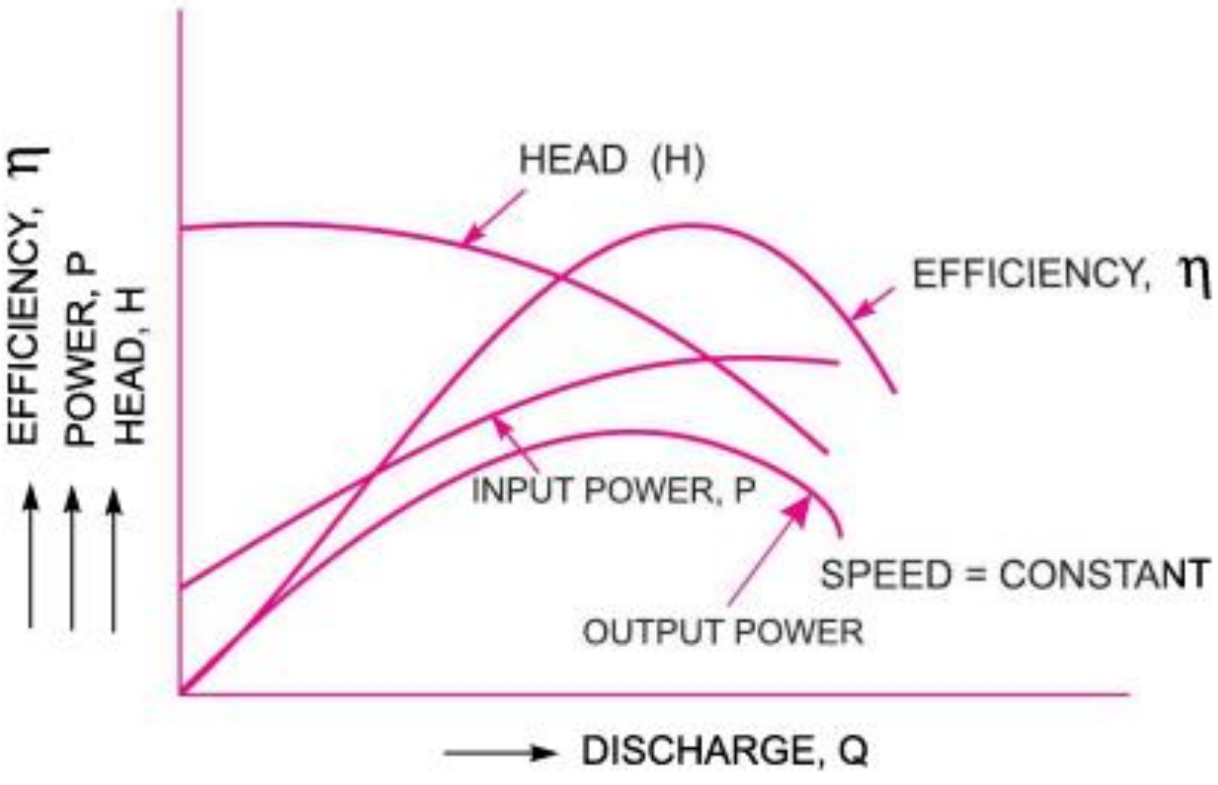


Fig. 19.15 *Operating characteristic curves of a pump.*

3.Constant Efficiency Curves : For obtaining constant efficiency curves for a **pump**, **the head** versus **discharge** curves and **efficiency** versus **discharge** curves for different speed are used. Figure

(a) shows the head versus discharge curves for different speeds.

- The efficiency versus discharge curves for the different speeds are as shown in Figure

(b). By combining these curves (**H~ Q curves** and **η ~ Q** curves), constant efficiency curves are obtained as shown in Figure

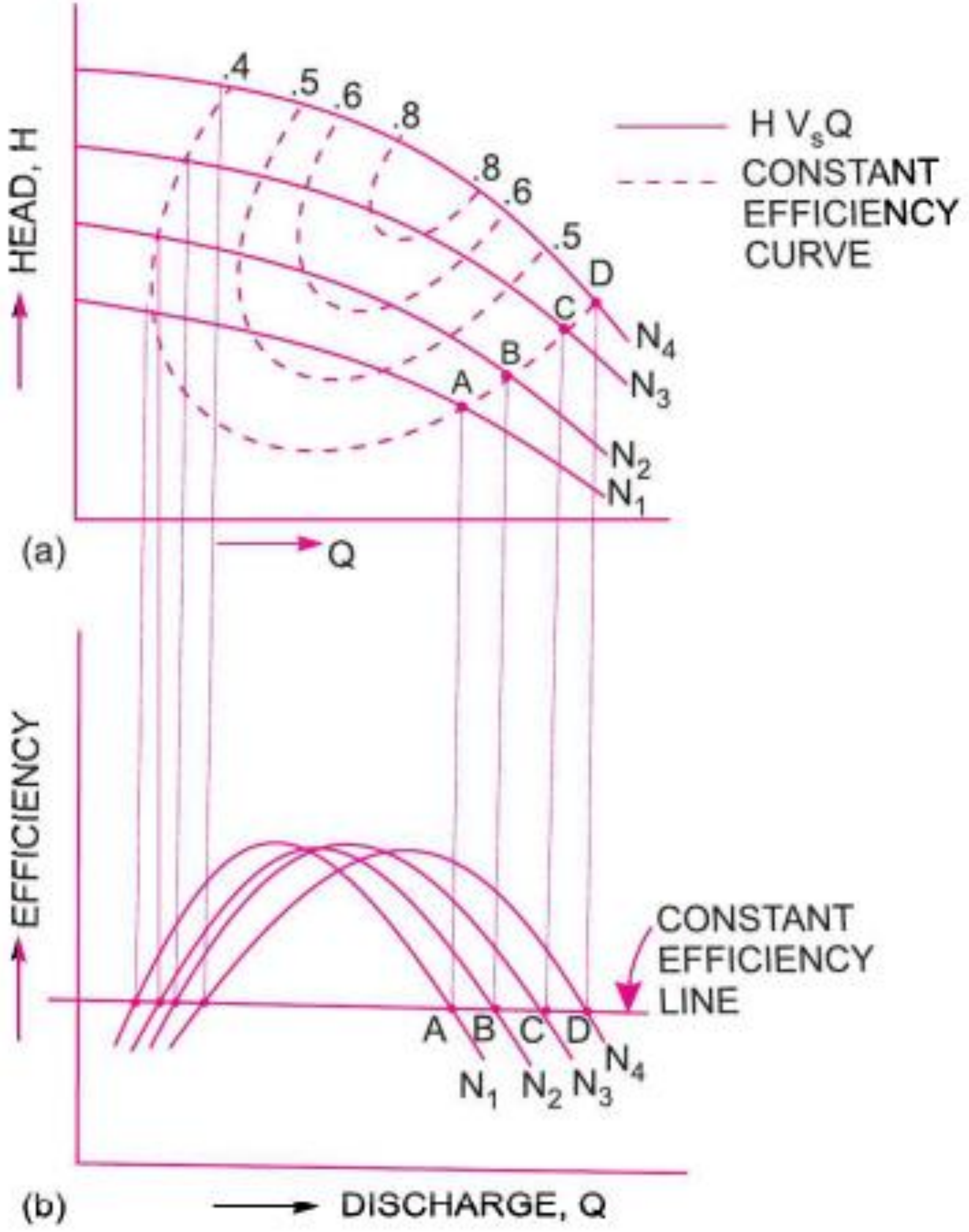


Fig. 19.16 *Constant efficiency curves of a pump.*

(a). For plotting **the constant efficiency curves** (also known as **iso-efficiency curves**), horizontal lines representing constant efficiencies are drawn on the **n~Q curves**. The points, at which these lines cut the efficiency curves at various speeds, are transferred to the corresponding **H~ Q curves.**

The points having the same efficiency are then joined by smooth curves. These smooth curves represents the **iso- efficiency curves.**

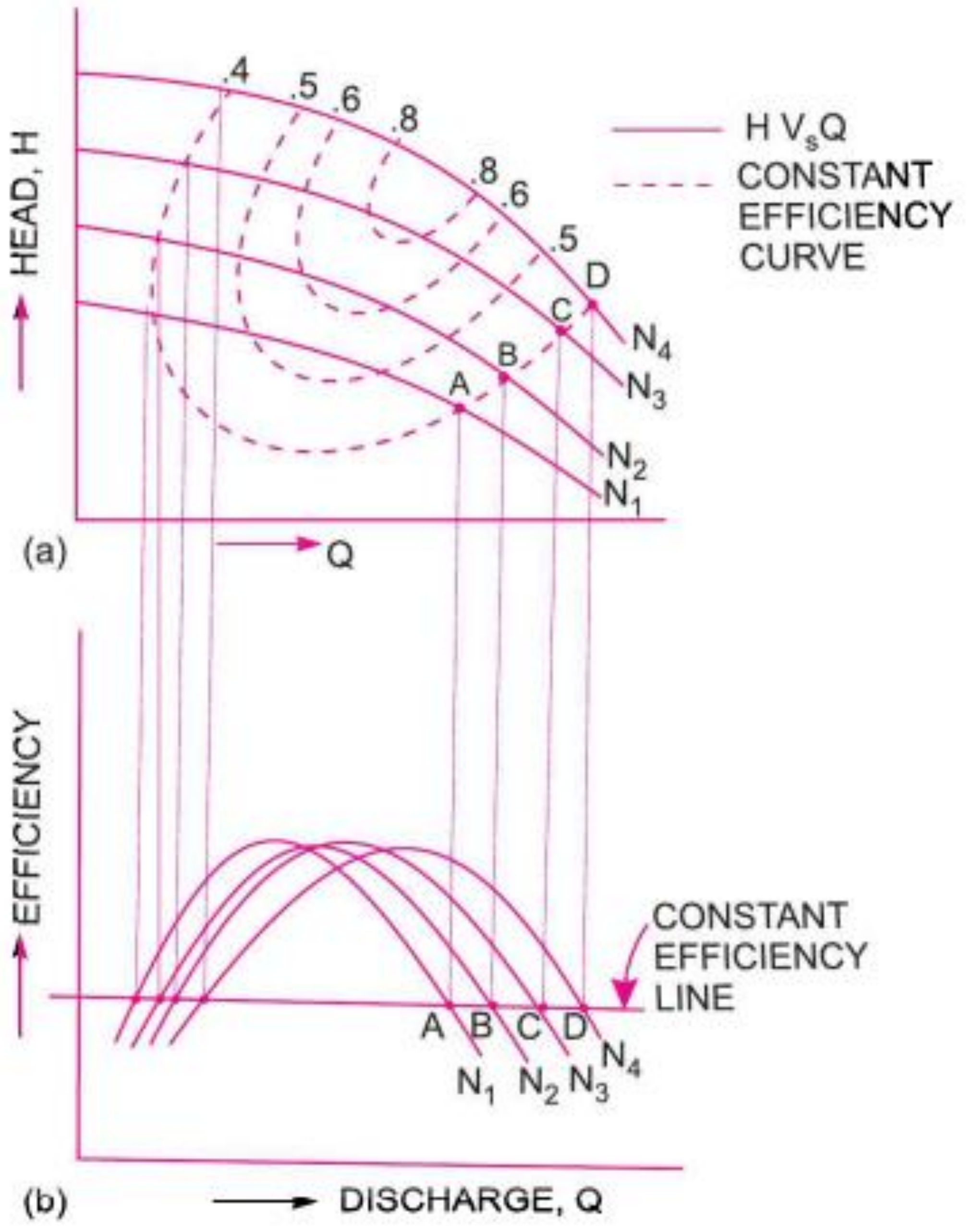
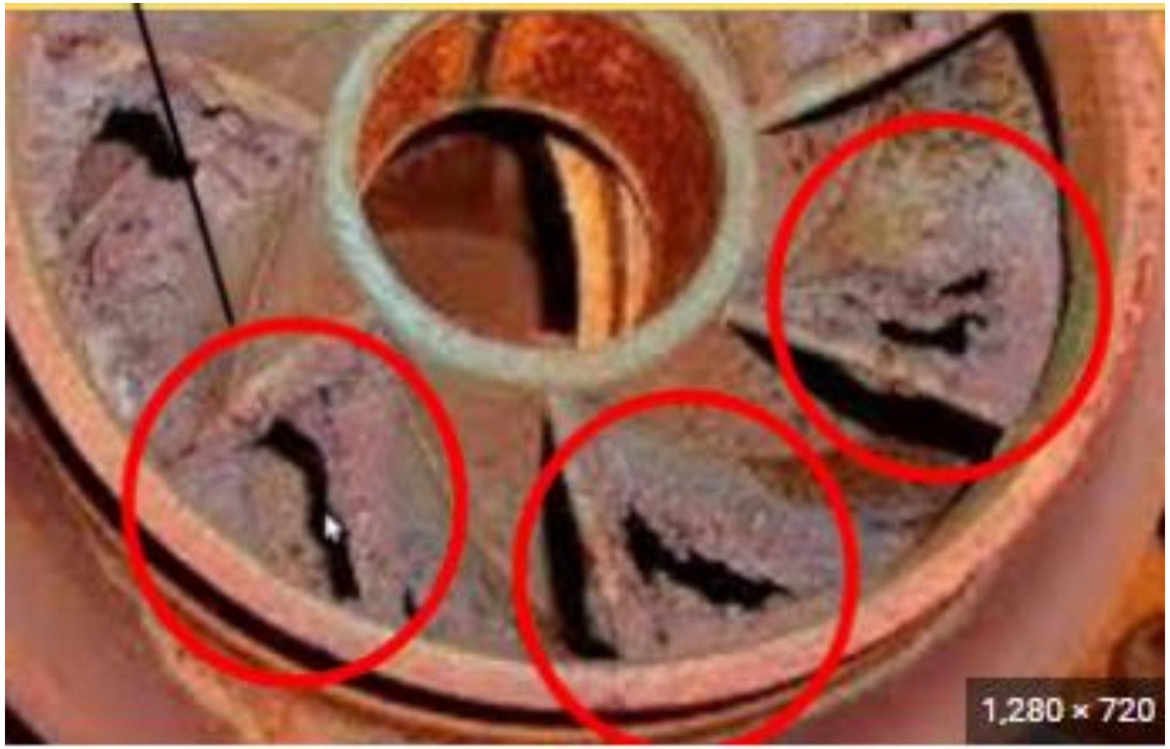


Fig. 19.16 *Constant efficiency curves of a pump.*

CAVITATION:

- Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created.



- The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed.

These vapour bubbles are carried along with the flowing liquid to **higher pressure zones** where **these vapours condense and bubbles collapse**. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to **high local stresses**. Thus the surfaces are damaged.

Precaution Against Cavitation:

The following precautions should be taken against cavitation :

(i) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its **vapour pressure**. If the flowing liquid is water, then the absolute pressure head **should not be below 2.5 m of water.**

(ii) The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation:

The following are the effects of cavitation :

(i) The metallic surfaces are damaged and cavities are formed on the surfaces.

(ii) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

(iii) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

- Hydraulic Machines Subjected to Cavitation. The hydraulic machines subjected to cavitation are reaction turbines and centrifugal pumps.

- **Cavitation in Turbines.** In turbines, only reaction turbines are subjected to cavitation.
- In reaction turbines the cavitation may occur at the **outlet of the runner** or at **the inlet of the draft- tube** where the pressure is considerably reduced (i.e., which may be below the vapour pressure of the liquid flowing through the turbine).
- Due to cavitation, the metal of the runner vanes and draft-tube is **gradually eaten away**, which results in lowering the efficiency of the turbine. Hence, the cavitation in a reaction turbine can be noted by a sudden drop in efficiency.
- In order to determine whether cavitation will occur in any portion of a reaction turbine,

The critical value of Thoma's cavitation factor (σ sigma) is calculated.

Thoma's Cavitation Factor for Reaction Turbines:

Prof. D. Thoma suggested a dimensionless number, called after his name Thoma's cavitation factor σ (sigma),

which can be used for determining the region where cavitation takes place in reaction turbines. The mathematical expression for the Thoma's cavitation factor is given by

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

where H_b = Barometric pressure head in m of water,

H_{atm} = Atmospheric pressure head in m of water,

H_v = Vapour pressure head in m of water,

H_s = Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

H = Net head on the turbine in m.

Cavitation in Centrifugal Pumps. In centrifugal pumps the cavitation may occur at the **inlet of the impeller of the pump, or at the suction side of the pumps**, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur.

- The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (**σ**) is calculated.

Thoma's Cavitation Factor for Centrifugal Pumps. The mathematical expression for Thoma's cavitation factor for centrifugal pump is given by

Thoma's Cavitation Factor for Centrifugal Pumps. The mathematical expression for Thoma's cavitation factor for centrifugal pump is given by

$$\sigma = \frac{(H_b) - H_S - h_{LS}}{H} = \frac{(H_{atm} - H_V) - H_S - h_{LS}}{H}$$

where H_{atm} = Atmospheric pressure head in m of water or absolute pressure head at the liquid surface in pump,

H_V = Vapour pressure head in m of water,

H_S = Suction pressure head in m of water,

h_{LS} = Head lost due to friction in suction pipe, and

H = Head developed by the pump.

This value of Thoma's cavitation factor (**σ**) is compared with critical cavitation factor ($\boldsymbol{\sigma_C}$) for that type of turbine pump. If the value of σ is greater than($\boldsymbol{\sigma_C}$), the cavitation will not occur in that turbine or pump.

The critical cavitation factor ($\boldsymbol{\sigma_C}$) may be obtained from tables or empirical relationships.

The following empirical relationships are used for obtaining the value of σ for different turbines :

For Francis turbines,

$$\sigma_c = 0.625\left(\frac{N_s}{380.78}\right)^2 \simeq 431 \times 10^{-8}\, Ns^2$$

For Propeller turbines,

$$\sigma_c = 0.28 + \left[\frac{1}{7.5}\left(\frac{N_s}{380.78}\right)^3\right]$$

In the above expressions N_s is in (r.p.m., kW, m) units. If N_s is in (r.p.m., h.p., m) units, the empirical relationships would be as follows :

For Francis turbines,

$$\sigma_c = 0.625 \left(\frac{N_s}{444}\right)^2$$

$$\simeq 317 \times 10^{-8} \times N_s^2$$

For Propeller turbines,

$$\sigma_c = 0.28 + \left[\frac{1}{7.5}\left(\frac{N_s}{444}\right)^3\right]$$

Problem 19.23 *A Francis turbine has been manufactured to develop 15000 horse power at the head of 81 m and speed 375 r.p.m. The mean atmospheric pressure at the site is 1.03 kgf/cm² and vapour pressure 0.03 kgf/cm². Calculate the maximum permissible height of the runner above the tail water level to ensure cavitation free operation. The critical cavitation factor for Francis turbine is given by*

$$\sigma_c = 317 \times 10^{-8} \times N_s^{\,2}$$

where N_s is the specific speed of the turbine in M.K.S. units.

Horse power developed, $P = 15000$

Head, $H = 81$ m

Speed, $N = 375$ r.p.m.

Atmospheric pressure, $p_a = 1.03 \text{ kgf/cm}^2 = 1.03 \times 9.81 \text{ N/cm}^2$

$= 1.03 \times 9.81 \times 10^4 \text{ N/m}^2$

∴ Atmospheric pressure head in meter of water,

$$H_{atm} = \frac{p_a}{\rho g} = \frac{1.03 \times 9.81 \times 10^4}{1000 \times 9.81} = 10.3 \text{ m}$$

Vapour pressure, $p_v = 0.03 \text{ kgf/cm}^2 = 0.03 \times 9.81 \text{ N/cm}^2 = 0.03 \times 9.81 \times 10^4 \text{ N/m}^2$

$\therefore$ Vapour pressure head in meter of water,

$$H_v = \frac{p_v}{\rho g} = \frac{0.03 \times 9.81 \times 10^4}{1000 \times 9.81} = 0.3 \text{ m}$$

Critical cavitation factor, $\sigma_c = 317 \times 10^{-8} \; N_s^2$

where N_s is the specific speed of the turbine in M.K.S. units

Now specific speed of turbine is given by

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \; : \; = \frac{375 \times \sqrt{15000}\,*}{81^{5/4}} = 189 \text{ r.p.m.}$$

Critical cavitation factor, $\sigma_c = 317 \times 10^{-8} \; N_s^2$

$$\sigma_c = 317 \times 10^{-8} \times 189^2 = 0.1132$$

Now let H_s = Suction pressure head in meter of water at the outlet of Francis turbine or height of the turbine runner above the tail water surface.

$$\sigma_c = \frac{H_{atm} - H_v - H_s}{H} \qquad 0.1132 = \frac{10.3 - 0.3 - H_s}{81}$$

$$0.1132 \times 81 = 10 - H_s \quad \text{or} \quad H_s = 10 - 0.1132 \times 81 = \mathbf{0.8308\ m.\ Ans.}$$

Hence, maximum permissible height is 0.83 m above the tail water level.

Maximum Suction Lift or Maximum suction height

Figure showing that Centrifugal pump that lift liquid from a sump

The free surface of liquid at a depth of h_s

The Liquid flowing through a pipe with a velocity V_S

Let h_s Suction head or lift

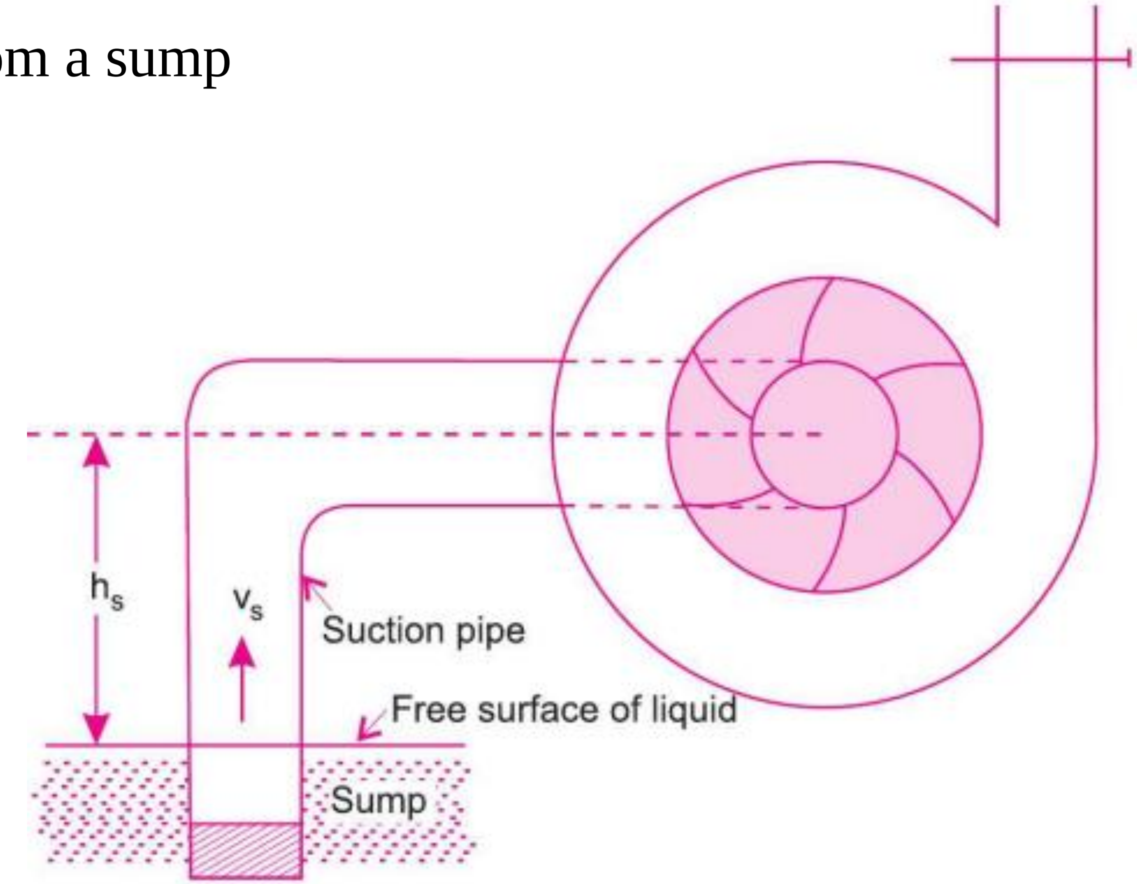


Fig. 19.17

Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

$$\frac{p_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L$$

p_a = Atmospheric pressure on the free surface of liquid,
V_a = Velocity of liquid at the free surface of liquid $\simeq 0$,
Z_a = Height of free surface from datum line = 0,
p_1 = Absolute pressure at the inlet of pump,
V_1 = Velocity of liquid through suction pipe = v_s,
Z_1 = Height of inlet of pump from datum line = h_s,
h_L = Loss of head in the foot valve, strainer and suction pipe = h_{fs}.

$$\frac{p_a}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

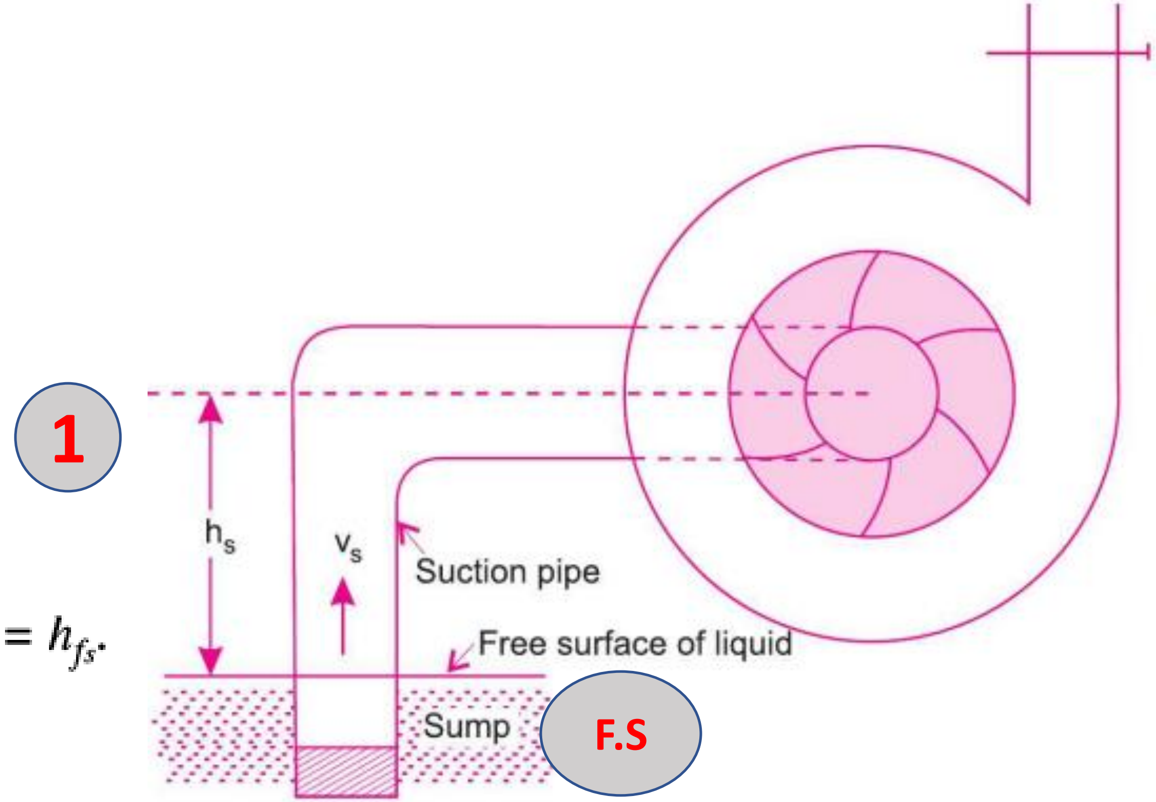


Fig. 19.17

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

For finding the maximum suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get

$p_1 = p_v$, where p_v = vapour pressure of the liquid in absolute units.

$$\frac{p_v}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\frac{p_a}{\rho g} = \frac{p_v}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$\frac{p_a}{\rho g}$ = Atmospheric pressure head = H_a (meter of liquid)

$\frac{p_v}{\rho g}$ = Vapour pressure head = H_v (meter of liquid)

$$H_a = H_v + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s}$$

$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s}$$

Equation (19.31) gives the value of maximum suction lift (or maximum suction height) for a centrifugal pump. Hence, the suction height of any pump should not be more than that given by equation (19.31). If the suction height of the pump is more, then vaporization of liquid at inlet of pump will take place and there will be a possibility of cavitation.

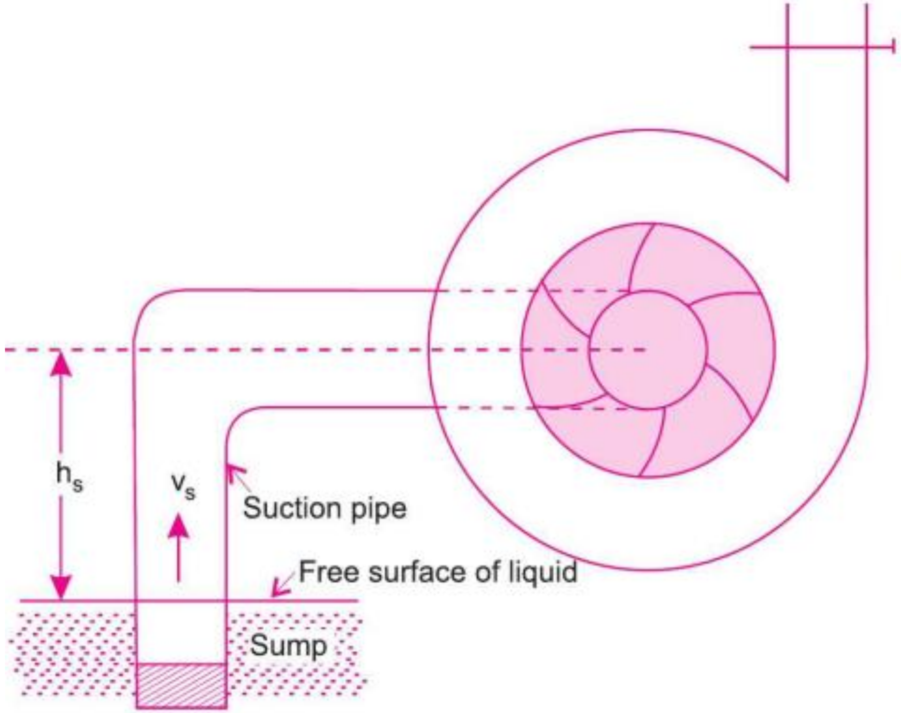


Fig. 19.17

NET POSITIVE SUCTION HEAD (NPSH)

The term NPSH (Net Positive Suction Head) is very commonly used in the pump industry. Actually the minimum suction conditions are more frequently specified in terms of NPSH.

The net positive suction head (NPSH) is defined as the *absolute* pressure head at the inlet to the pump, minus the vapour pressure head (in absolute units) plus the velocity head.

∴ NPSH = Absolute pressure head at inlet of the pump – vapour pressure head (absolute units) + velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\text{NPSH} = \left[\frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s} \quad = \left[\left(H_a - h_s - h_{f_s}\right) - H_v\right]$$

$\frac{p_a}{\rho g} = H_a$ = Atmospheric pressure head,

$\frac{p_v}{\rho g} = H_v$ = Vapour pressure head

NSPH $= \left[\left(H_a - h_s - h_{f_s}\right) - H_v\right]$

The right hand side of equation (19.33) is the total suction head. Hence NPSH is equal to total suction head. Thus NPSH may also be defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

For any pump installation, a distinction is made between the required NPSH and the available NPSH. The value of required NPSH is given by the pump manufacturer. This value can also be determined experimentally. For determining its value, the pump is tested and minimum value of h_s is obtained at which the pump gives maximum efficiency without any objectional noise (*i.e.*, cavitation free). The required NPSH varies with the pump design, speed of the pump and capacity of the pump.

When the pump is installed, the available NPSH is calculated from equation (19.33). In order to have cavitation free operation of centrifugal pump, the available NPSH should be greater than the required NPSH.

CAVITATION IN CENTRIFUGAL PUMP

Thoma's cavitation factor is used to indicate whether cavitation will occur in pumps. Equation (19.24) gives the value of Thoma's cavitation factor for pumps as

$$\sigma = \frac{(H_{atm} - H_v) - H_s - h_{f_s}}{H}$$

$$= \frac{H_a - H_v - h_s - h_{f_s}}{H_m} \quad (\because\ H_s = h_s \text{ and } h_{L_s} = h_{f_s})\ (H = H_m \text{ for pumps})$$

$$H_a - H_v - h_s - h_{f_s} = \text{NPSH}$$

$$\sigma = \frac{\text{NPSH}}{H_m}$$

The following empirical relation is used to determine the value of σ_c.

$$\sigma_c = 0.103 \left(\frac{N_s}{1000}\right)^{4/3}$$

$$= 0.103 \frac{N_s^{4/3}}{(10^3)^{4/3}} = \frac{0.103\, N_s^{4/3}}{10^4}$$

$$= 1.03 \times 10^{-3}\, N_s^{4/3}$$

Reciprocating pumps

MAIN PARTS IN RECIPROCATING PUMP

1. A cylinder with a piston, piston rod, connecting rod and a crank

2. Suction pipe

3. Delivery pipe

4. Suction valve

5. Delivery valve.

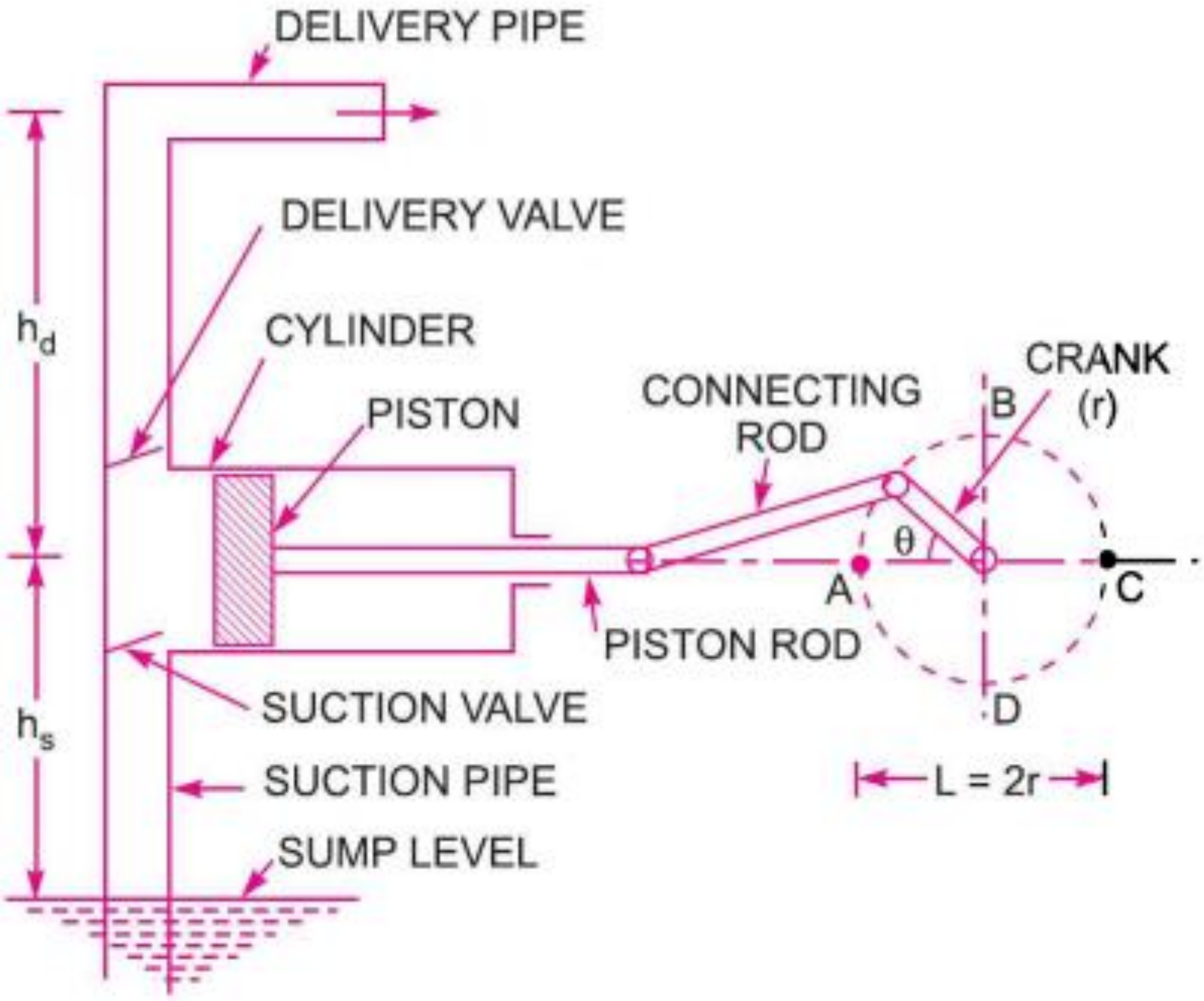


Fig. 20.1 *Main parts of a reciprocating pump.*

- Figure shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder

- The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor.

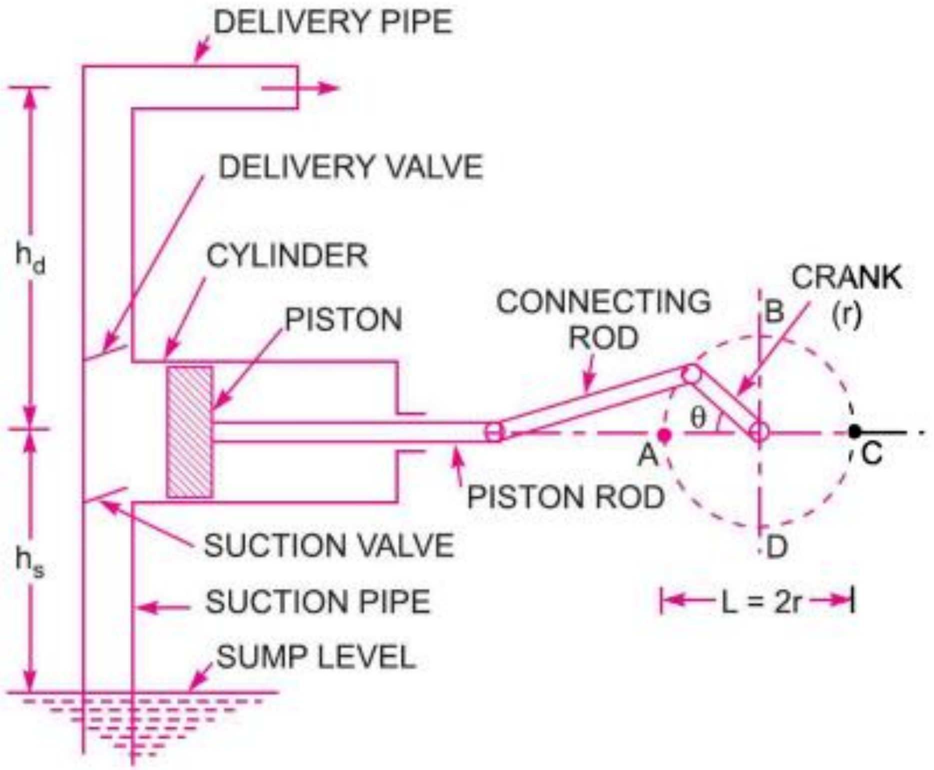


Fig. 20.1 *Main parts of a reciprocating pump.*

- Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder.

- The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only.

- Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

- When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at **A,** the piston is at the extreme left position in the cylinder.
- As the crank is rotating from A to C, (i.e., from $\theta = 0°$ t0 $\theta = 180°$), the piston is moving towards right in the cylinder.
- The movement of the piston towards right creates a **partial vacuum in the cylinder**. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder.

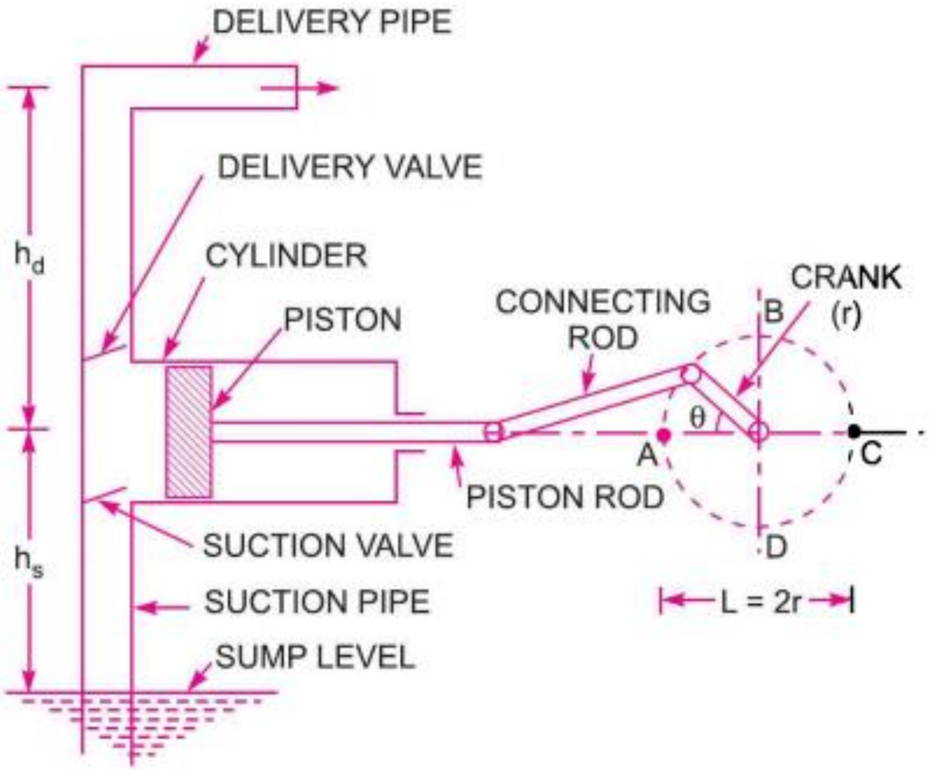


Fig. 20.1 *Main parts of a reciprocating pump.*

Thus, the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

- When crank is rotating from C to A (i.e., from θ = 180° to = 360°), the piston from its extreme right position starts moving towards left in the cylinder.

- The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure.

- Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

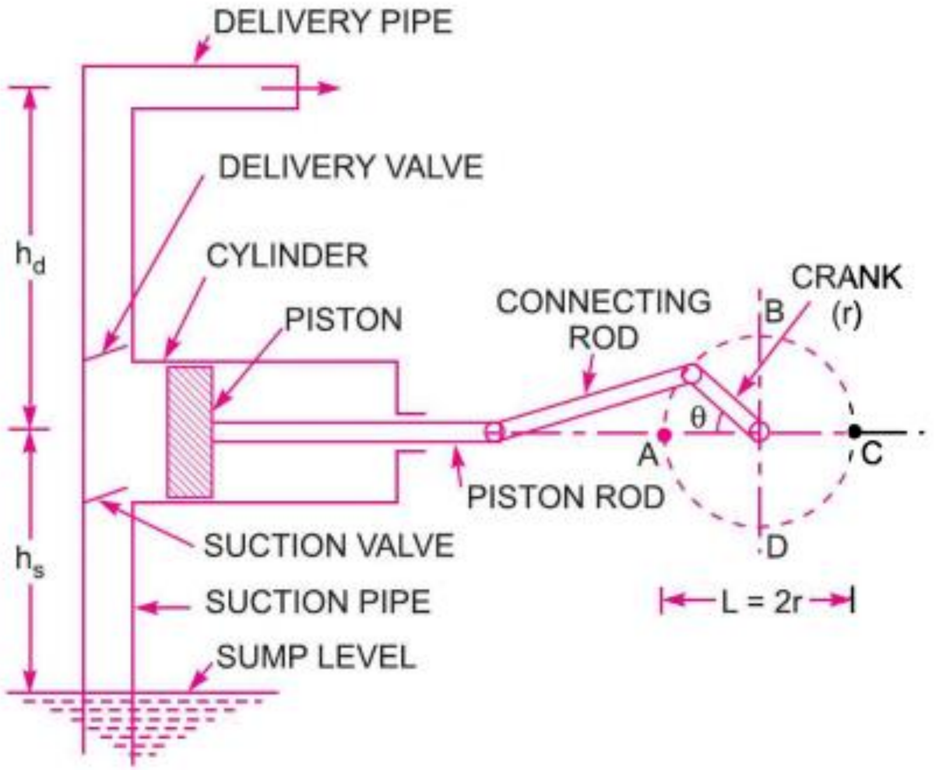


Fig. 20.1 *Main parts of a reciprocating pump.*

Discharge through a reciprocating pump

Consider a single cylinder acting reciprocation pump

D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$= \frac{\pi}{4} D^2$$

r = Radius of crank

L = Length of the stroke = $2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

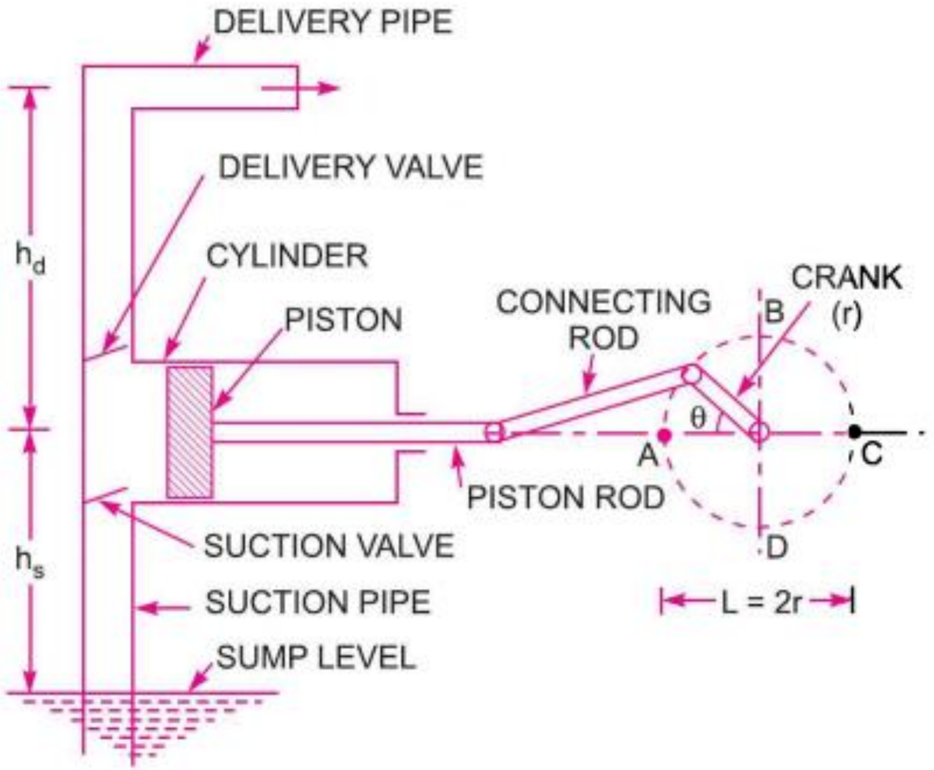


Fig. 20.1 *Main parts of a reciprocating pump.*

Volume of water delivered in one revolution or discharge of water in one revolution

$$= \text{Area} \times \text{Length of stroke} = A \times L$$

Number of revolution per second, $= \dfrac{N}{60}$

Discharge of the pump per second,

$$Q = \text{Discharge in one revolution} \times \text{No. of revolution per second}$$

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60}$$

Weight of water delivered per second,

$$W = \rho \times g \times Q = \frac{\rho g ALN}{60}.$$

Work done by the reciprocating pump

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d)$$

where $(h_s + h_d)$ = Total height through which water is lifted.

Weight, W, is given by

$$W = \frac{\rho g \times ALN}{60}.$$

$$\text{Work done per second} = \frac{\rho g \times ALN}{60} \times (h_s + h_d)$$

∴ Power required to drive the pump, in kW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} \text{ kW}$$

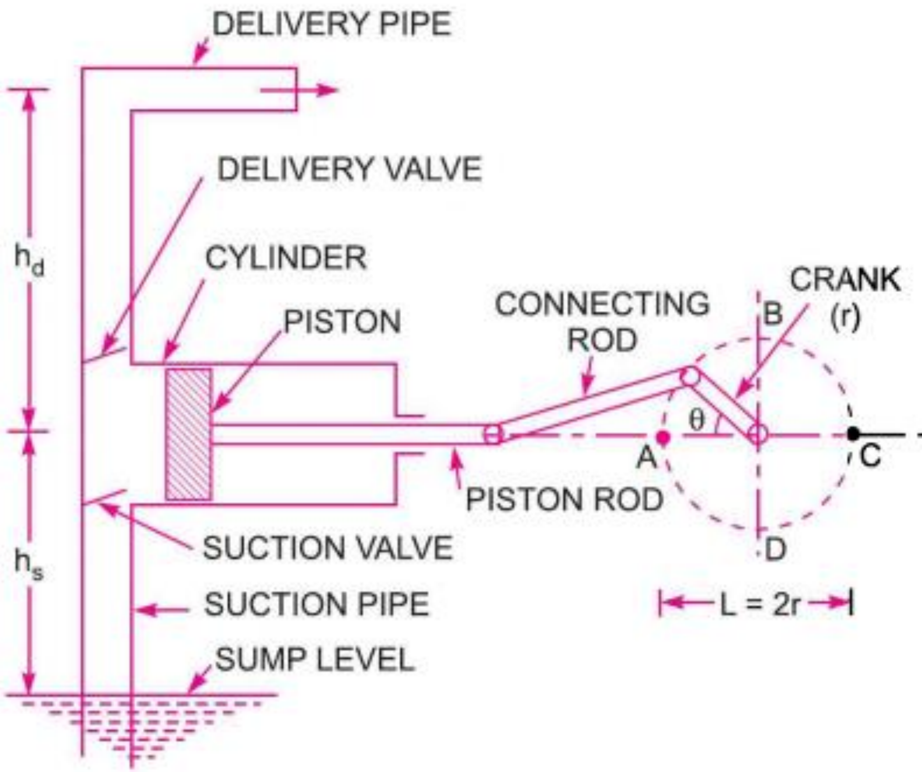


Fig. 20.1 *Main parts of a reciprocating pump.*

SLIP OF RECIPROCATING PUMP

Slip of a pump is defined as the difference between the **theoretical discharge** and **actual discharge of the pump**

- The discharge of a single-acting pump given by equation and are theoretical discharge. The actual discharge of a pump is less theatre theoretical discharge due to leakage.
- The difference of the theoretical discharge and actual discharge is known as slip of the pump. Hence, mathematically.

$$\text{Slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

where C_d = Co-efficient of discharge.

$$= (1 - C_d) \times 100$$

Negative Slip of the Reciprocating Pump.

- Slip is equal to the difference of theoretical discharge and actual discharge.
- If actual discharge is more than the theoretical discharge, the slip of the pump will become **~ve.**
- In that case, the slip of the pump is known as **negative slip.** Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at highspeed.

CLASSIFICATION OF RECIPROCATING PUMPS

The reciprocating pumps may be classified as :

1. According to the water being in contact with one side or both sides of the piston, and
2. According to the number of cylinders provided.

If the water is in contact with one side of the piston, the pump is known as single-acting. On the other hand, if the water is in contact with both sides of the piston, the pump is called double-acting. Hence,

Classification according to the contact of water is:

(i) Single-acting pump, and

(ii) Double-acting pump.

CLASSIFICATION OF RECIPROCATING PUMPS

According to the number of cylinder provided, the pumps are classified as :

(i) Single cylinder pump,

(iii) Triple cylinder pump.

(ii) Double cylinder pump, and

Problem 20.1 *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*

(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01\ m^3/s$

Dia. of piston, $D = 200\ mm = .20\ m$

$\therefore$ Area, $A = \dfrac{\pi}{4}(.2)^2 = .031416\ m^2$

Stroke, $L = 400\ mm = 0.40\ m.$

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047\ m^3/s.\ Ans.}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955.\ Ans.}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047\ m^3/s.\ Ans.}$$

And percentage slip

$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%.\ Ans.}$$

VARIATION OF VELOCITY AND ACCELERATION IN THE SUCTION AND DELIVERY PIPES DUE TO ACCELERATION OF THE PISTON

- We know that when crank starts rotating, the piston moves forwards and backwards in the cylinder. At the extreme left position and right position of the piston in the cylinder, the velocity of the piston is zero.
- The velocity of the piston is maximum at the centre of the cylinder.
- This means that at the start of a stroke (may be suction or delivery stroke), the velocity of the piston is **zero** and this velocity becomes **maximum** at the centre of each stroke and again becomes **zero** at the end of each stroke.
- Thus at the beginning of each stroke, the piston will be having an acceleration and at the end of each stroke, the piston will be having a retardation.

- The water in the cylinder is in contact with the piston and hence the water, flowing from the suction pipe or to the delivery pipe will have an acceleration at the beginning of each stroke and a retardation at the end of each stroke.
- This means the velocity of flow of water in the suction and delivery pipe will not be uniform.
- Hence, an accelerative or retarding head will be acting on the water flowing through the suction or delivery pipe. This accelerative or retarding head will change the pressure inside the cylinder.
- If the ratio of length of connecting rod to the radius of crank (i.e., L/r) is very large, then the motion of the piston can be assumed as simple harmonic in nature.

Figure shows the cylinder of a reciprocating single-acting pump, fitted with a piston which is connected to the crank.

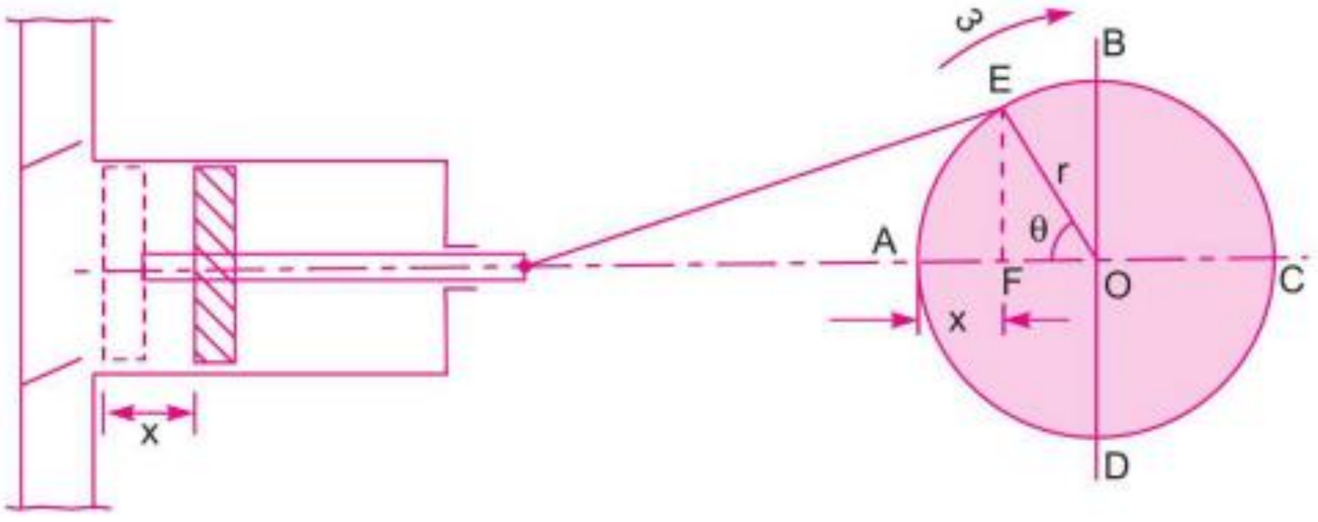


Fig. 20.3 *Velocity and acceleration of piston.*

Let the crank is rotating at a constant angular speed.

Let ω = Angular speed of the crank in rad./s,

A = Area of the cylinder,

a= Area of the pipe (suction or delivery),

l= Length of the pipe (suction or delivery), and

r= Radius of the crank.

In the beginning, the crank is at A (which is called inner dead centre) and the piston in the cylinder is at a position shown by dotted lines.

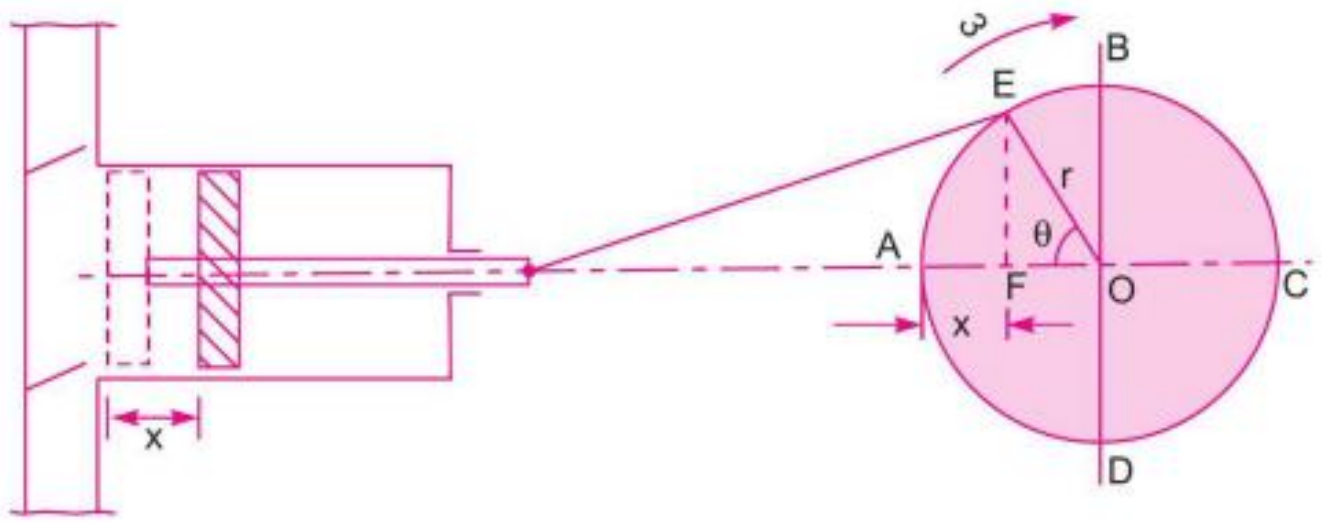


Fig. 20.3 *Velocity and acceleration of piston.*

The crank is rotating with an angular velocity **ω** and let in time **'t'** seconds, the crank turns through an angle θ (in radians) from A

The displacement of the piston in time 't' is 'x' as shown in Figure

Now θ = Angle turned by crank in radian in time 't'

$= \omega t$

The distance travelled by the piston is given as

X = Distance AF = AO – FO

X = Distance AF = AO – FO

$= r - r\cos\theta$

$$\mathbf{X = r - r\cos\omega t} \quad \longrightarrow \quad a$$

The velocity of the piston obtained by diffracting equation (ii)with respect to ‘t’

Velocity of the piston

$$= v = \frac{dx}{dt} = \frac{d}{dt}(\bar{r} - r\cos(\omega t))$$

$$= 0 - r[-\sin\omega t] \times \omega$$

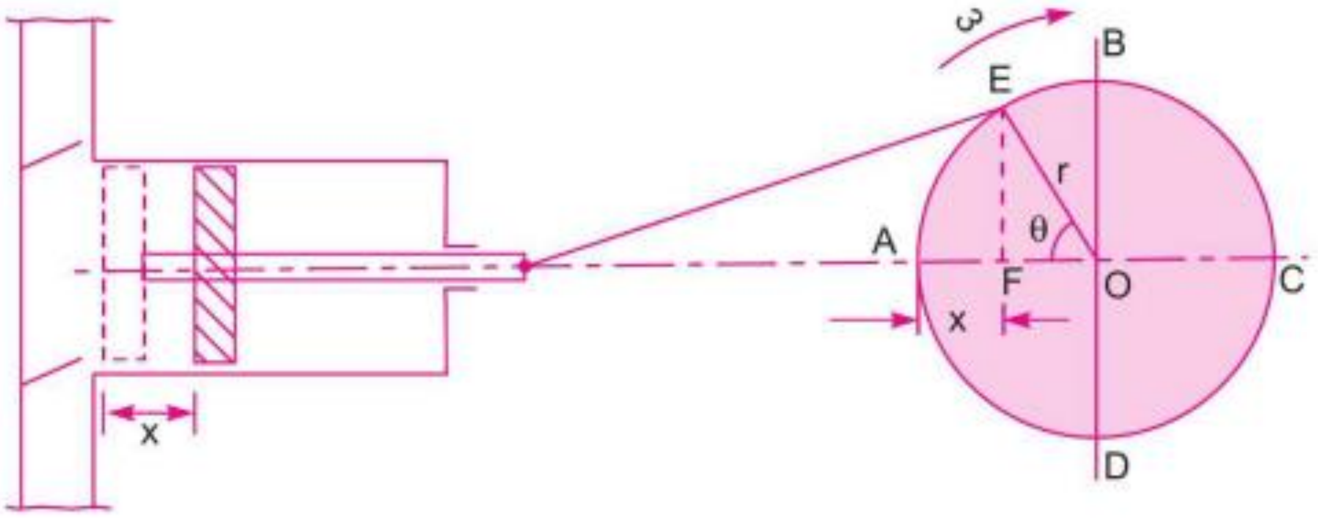


Fig. 20.3 *Velocity and acceleration of piston.*

Now from the continuity equation,

the volume of the water flowing into the cylinder per second is equal to the volume of water flowing from the pipe per second

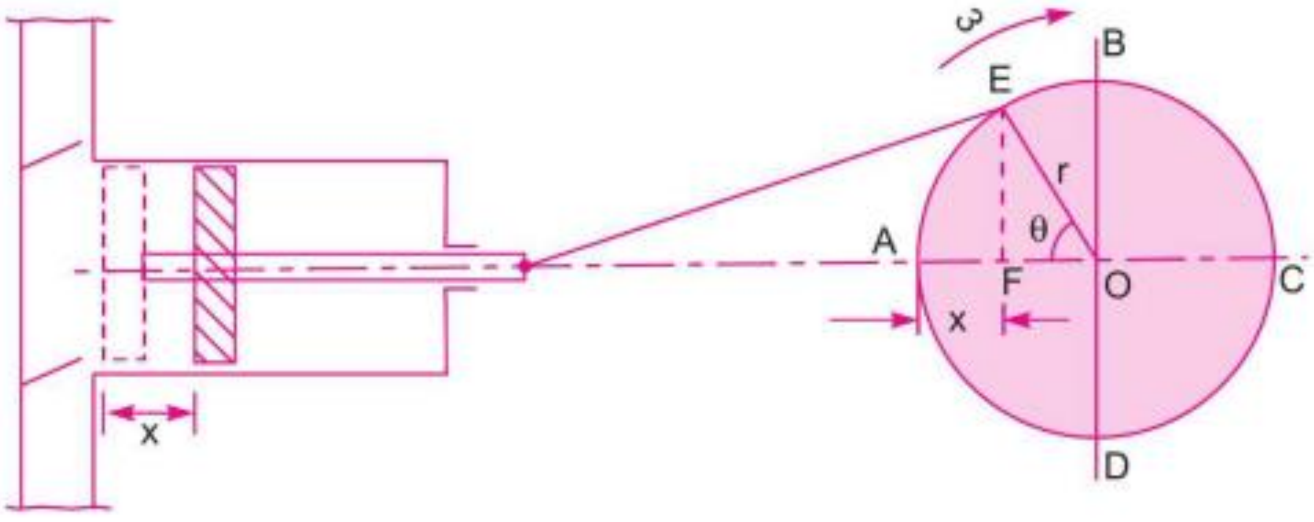


Fig. 20.3 *Velocity and acceleration of piston.*

Velocity of water per second × Area of cylinder

= Velocity of water in pipe × Area of the pipe

$V \times A = v \times a$ (V = velocity of water in cylinder = velocity of piston)

Where v = velocity of water in pipe

$$v = \frac{v \times A}{a}$$

$$V = \frac{A}{a} \ \omega r \sin \omega t$$

The acceleration of water in pipe is obtained by differentiating **VELOCITY** equation with respect to the 't'

Acceleration of water in pipe

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a}\ \omega r \sin \omega t\right)$$

Acceleration $a = \frac{A}{a}\omega^2$ r cos ωt

Mass of the water in pipe = ρ × Volume of water in pipe

$= \rho \times (Area\ of\ the\ pipe\ \times Length\ of\ the\ pipe\ )$

$= \rho \times (a\ \times l\ )$

Force required to accelerate water in the pipe = Mass of water in pipe × acceleration of water in pipe

Force required to accelerate water in the pipe $= \rho a l \times \frac{A}{a}\omega^2$ r cos ωt

Intensity of pressure due to acceleration $= \frac{Force\ requared\ to\ accelrate\ the\ pipe}{Area\ of\ pipe}$

$$= \frac{\rho a l \times \frac{A}{a} \omega^2 \text{ r cos ωt}}{a}$$

$$= \rho l \times \frac{A}{a} \omega^2 \text{ r cos ωt}$$

Pressure head due to acceleration $\boldsymbol{h_a}$ due to acceleration

$$h_a = \frac{Intensity\ of\ prerssure\ due\ to\ acceleration}{Weight\ density\ of\ the\ water}$$

$$= \frac{\rho l \times \frac{A}{a} \omega^2 \text{ r cos } \theta}{\rho g}$$

$$h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 \text{ r cos } \theta$$

Pressure head due to acceleration $\boldsymbol{h_a}$ due to acceleration

$$h_a = \frac{l}{g} \times \frac{A}{a}\, \omega^2\, \mathrm{r}\cos\theta \quad \rightarrow \quad a$$

The pressure head due to acceleration in the suction and delivery pipes obtained from the equation by using subscripts 's and 'd 'as

$$h_s = \frac{l_s}{g} \times \frac{A}{a_s}\, \omega^2\, \mathrm{r}\cos\theta$$

$$h_d = \frac{l_d}{g} \times \frac{A}{a_d}\, \omega^2\, \mathrm{r}\cos\theta$$

The pressure value h_a due to acceleration Varies with the value of θ

when $\theta = 0^0$ $\quad$ $\boldsymbol{h_a = \frac{l}{g} \times \frac{A}{a}\, \omega^2\, \mathrm{r}}$

when $\theta = 90^0$ $\quad$ $\boldsymbol{h_a = 0}$

when $\theta = 180^0$ $\quad$ $h_a = -\frac{l}{g} \times \frac{A}{a}\, \omega^2\, \mathrm{r}$

Maximum pressure head due to acceleration

$$\boldsymbol{h_{max} = \frac{l}{g} \times \frac{A}{a}\, \omega^2\, \mathrm{r}}$$

INDICATOR DIAGRAM

- The indicator diagram for a reciprocating pump is defined as the graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank.

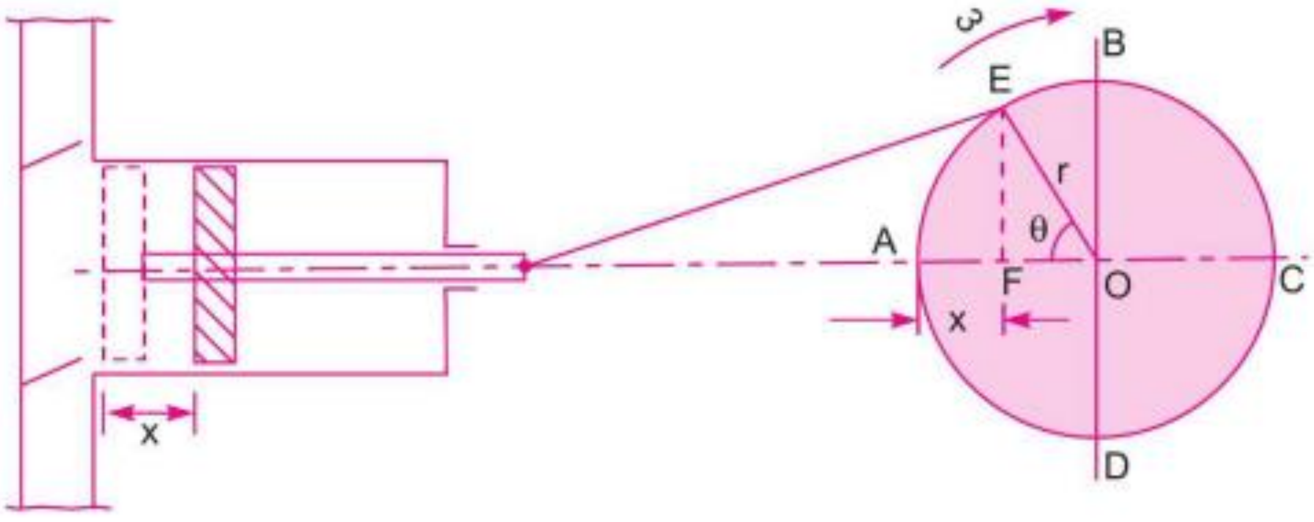
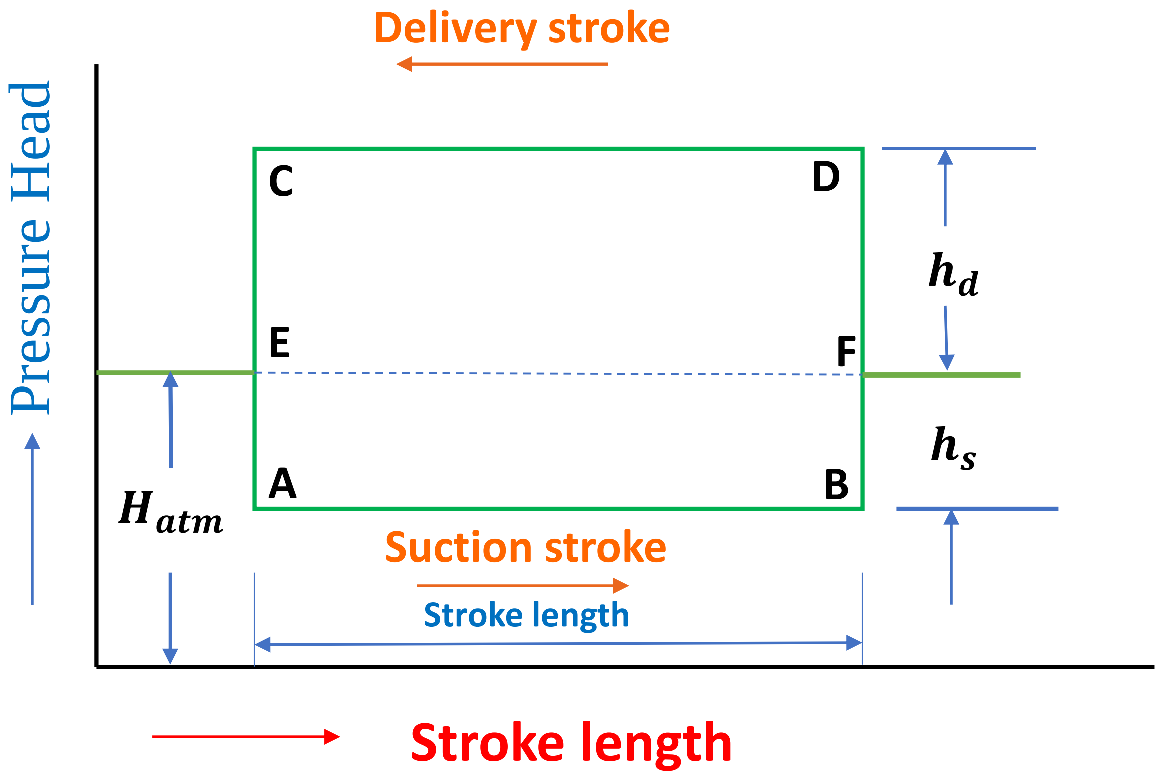


Fig. 20.3 *Velocity and acceleration of piston.*

- As the maximum distance travelled by the piston is equal to the stroke length and hence the indicator diagram is a graph between pressure head and stroke length of the piston for one complete revolution.

- The pressure head is taken as ordinate and stroke length as abscissa.

The graph between pressure head in the cylinder and stroke length of the piston for one complete revolution of the crank under ideal conditions is known as ideal indicator diagram.



In which line **EF** represents the atmospheric pressure head equal to 10.3 m of water.

H_{atm} = Atmospheric pressure head

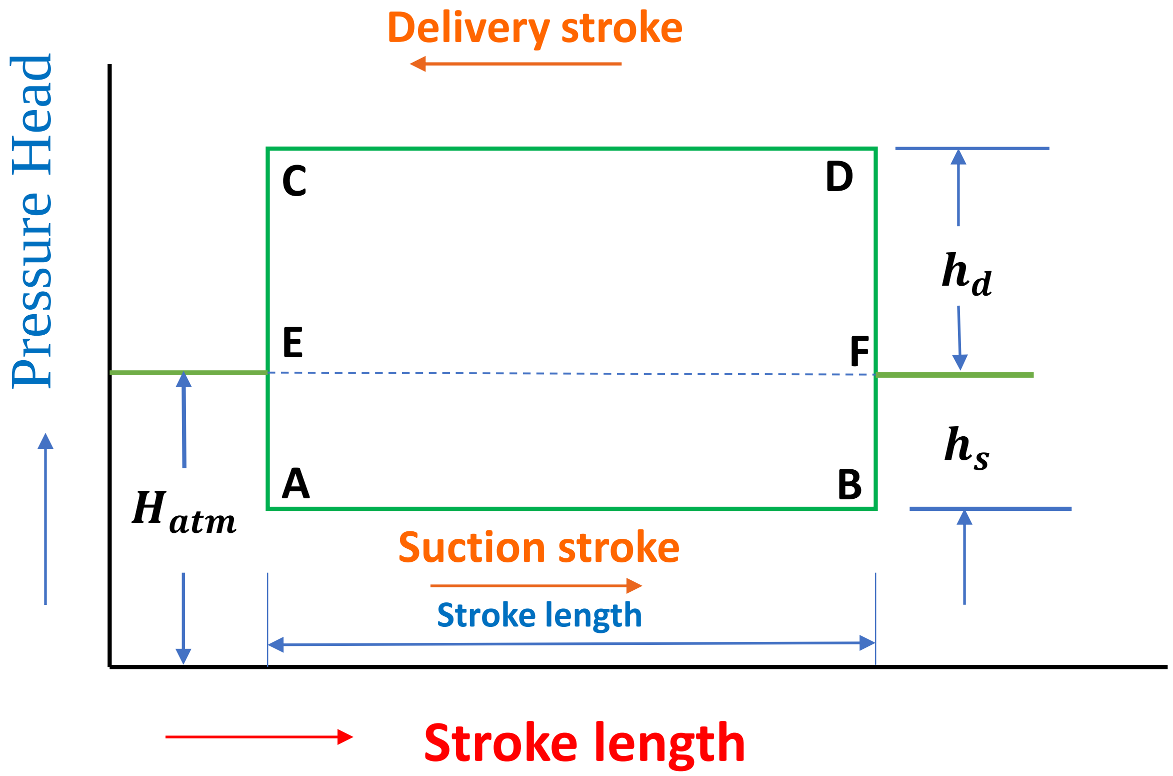
=10.3 m of water,

L = Length of the stroke,

h_s = Suction head, and

h_d = Delivery head.

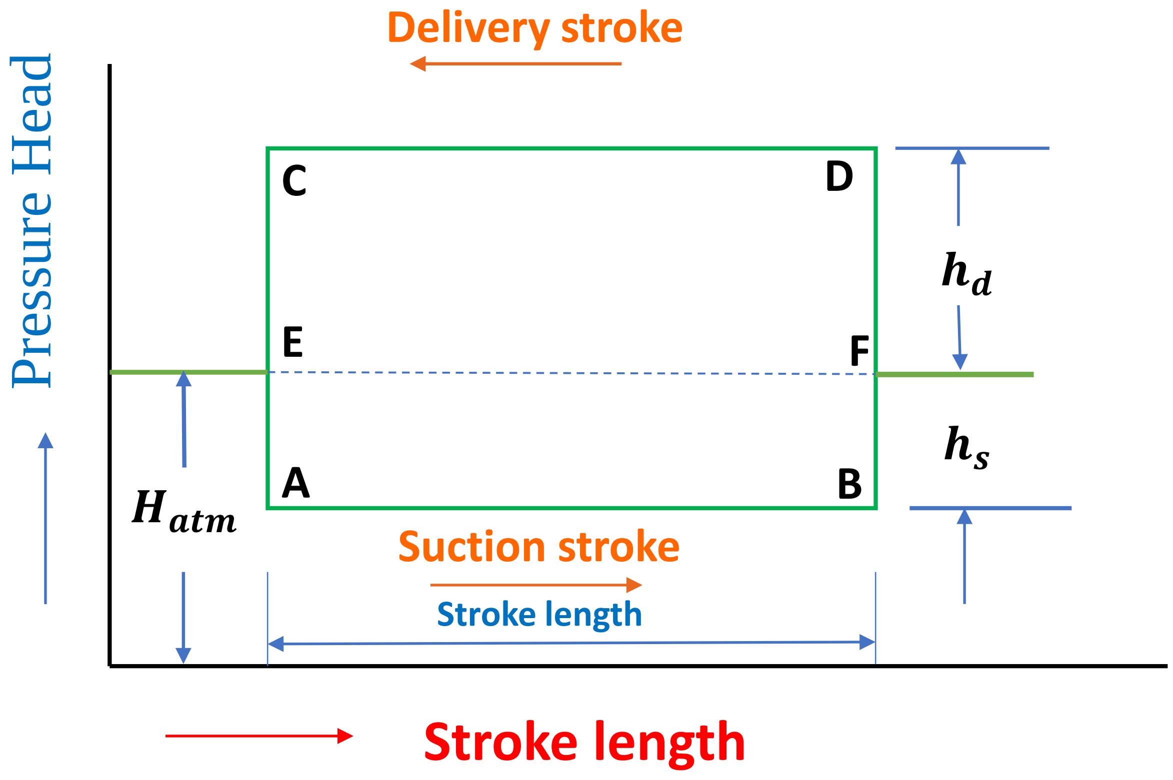
- During suction stroke, the pressure head in the cylinder is constant and equal to suction head (h_s),
- which is below the atmospheric pressure head (H_{atm}) by a height of h_s
- The pressure head during suction stroke is represented by a horizontal line AB which is below the line EF by a height of 'h_s,".



- During delivery stroke, the pressure head in the cylinder is constant and equal to delivery head (h_d)

- which is above the atmospheric head (H_{atm}) by a height of (h_d).

- Thus, the pressure head during delivery stroke is represented by a horizontal line CD which is above the line EF by a height of h_d.



Thus, for one complete revolution of the crank, the pressure head in the cylinder is represented by the diagram **A-B-C-D-A.** This diagram is known as **ideal indicator diagram.**

we know that the work done by the pump per second

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L(h_s + h_d) \qquad \left(\text{where } K = \frac{\rho g A N}{60} = \text{Constant}\right)$$

$$\propto L \times (h_s + h_d)$$

, area of indicator diagram

$$= AB \times BC$$

$$= AB \times (BF + FC)$$

$$= L \times (h_s + h_d).$$

Work done by pump $\propto$ Area of indicator diagram.

Effect of acceleration in suction and delivery pipes on Indicator diagram

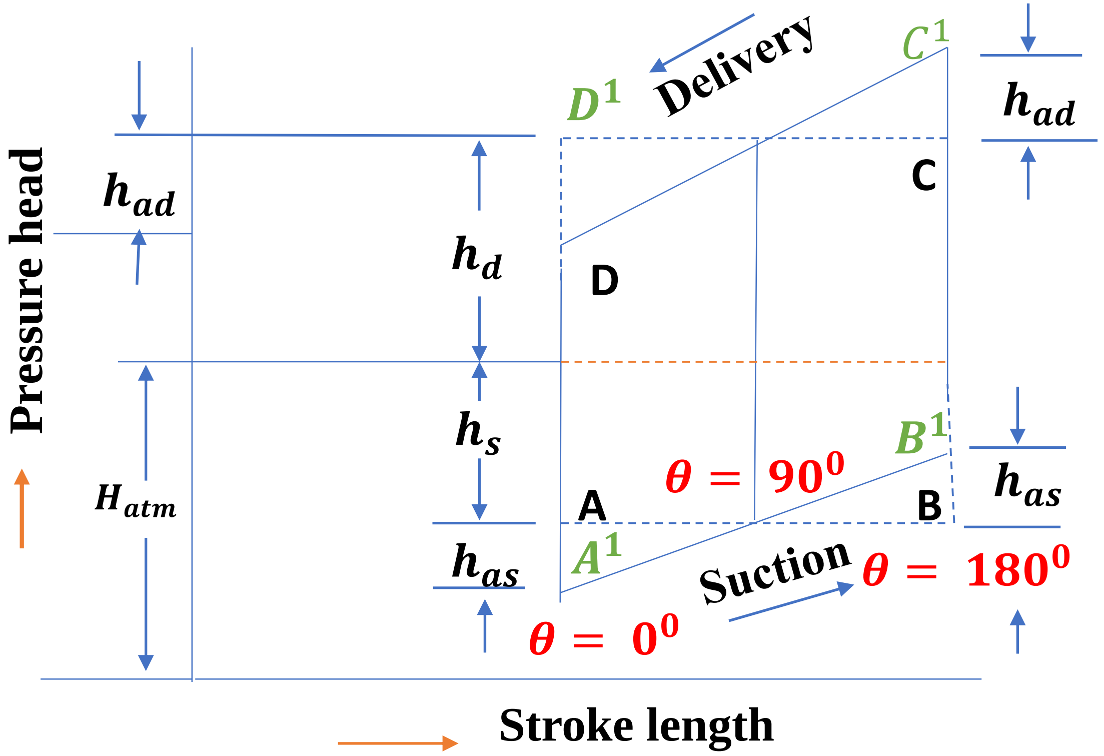
The pressure head due to acceleration in the suction pipe is given by equation

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$$

When $\theta = 0°$, $\cos\theta = 1$, and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

When $\theta = 90°$, $\cos\theta = 0$, and $h_{as} = 0$

When $\theta = 180°$, $\cos\theta = -1$, and $h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$.

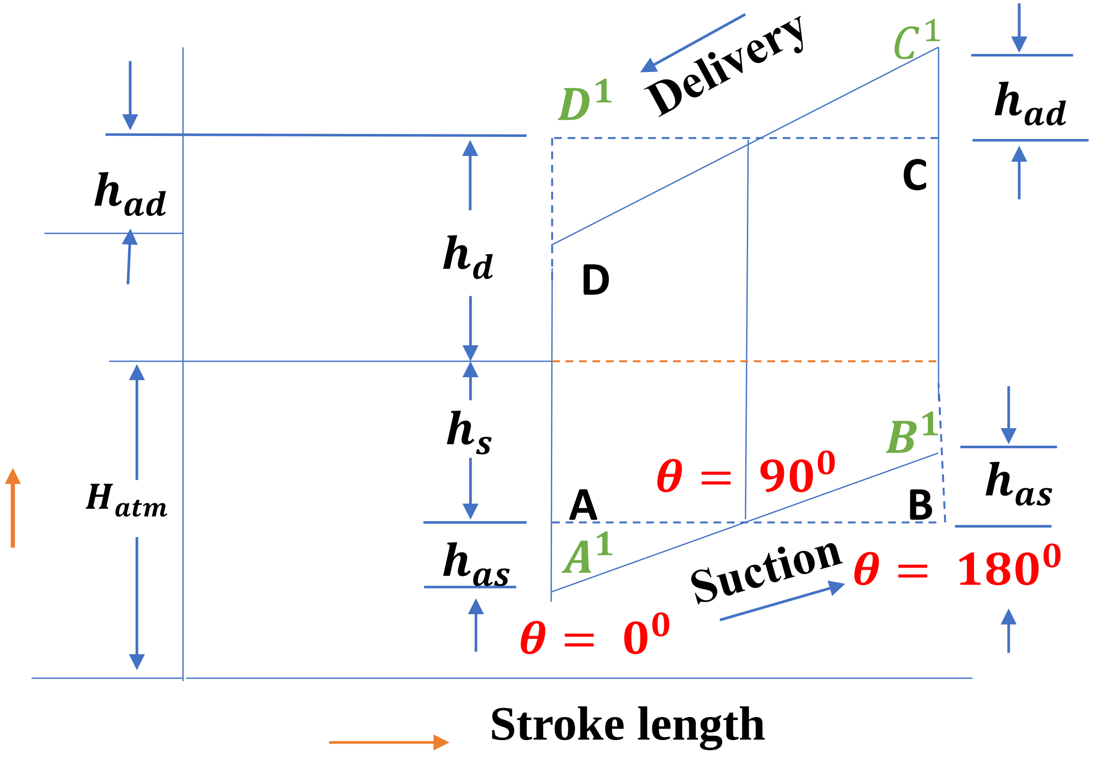


Thus, the pressure head inside the cylinder during suction stroke will not be equal to h_s as was the case for ideal indicator diagram,

but it will be equal to the sum of h_s and h_{as}

- At the beginning of suction stroke **θ = 0°, h_{as}** is **positive** and hence the pressure head will be **(h_s + h_{as})** below the atmospheric pressure head

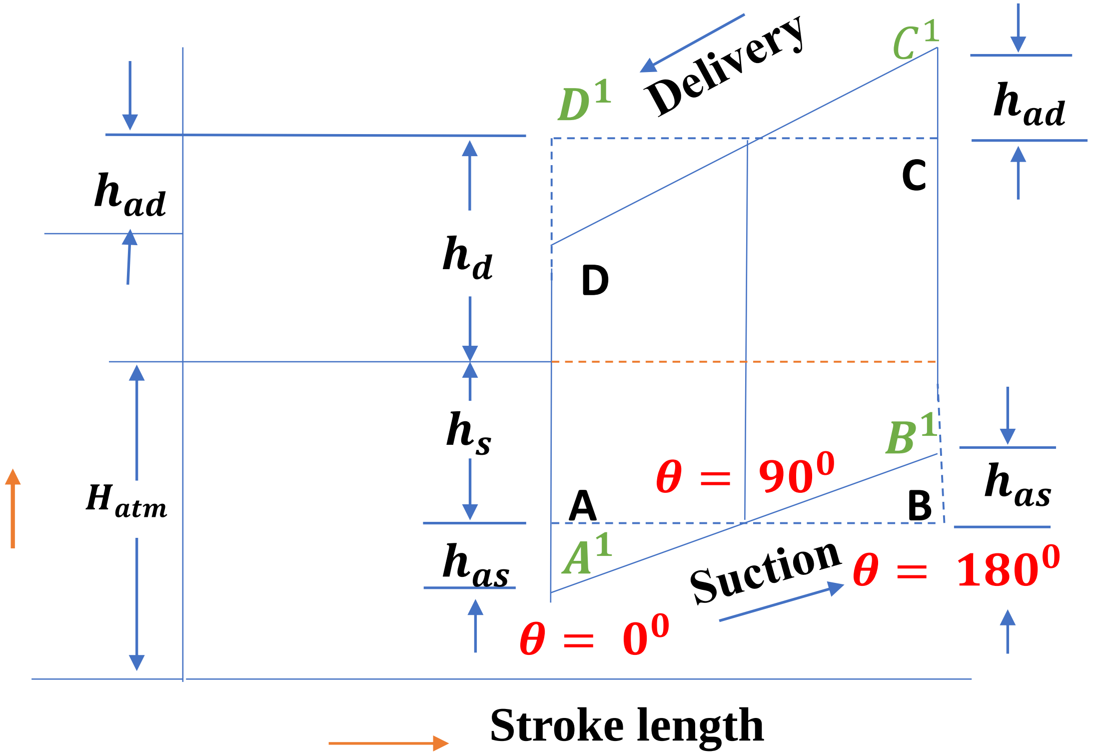
- At the middle of suction stroke **θ = 90°** and **h_{as} = 0** and hence pressure head in the cylinder will be **h_s**, below the atmospheric pressure head



- At the end of suction stroke, **θ = 180°** and **h_{as} is negative** and hence the pressure head in the cylinder will be **(h_s - h_{as})** below the atmospheric pressure head.

For suction stroke, the indicator diagram will be shown by A'GB'. Also the area of A' AG = Area of BGB'.

Similarly, the indicator diagram for the delivery stroke can be drawn.



- At the beginning of delivery stroke h_{ad} is positive and hence the hence the pressure head in the cylinder will be (h_d+ h_{ad}) above the atmospheric pressure head.

- At the middle of the delivery stroke, h_{ad} = 0 and hence pressure head in the cylinder is equal to h_d , above the atmospheric pressure head.

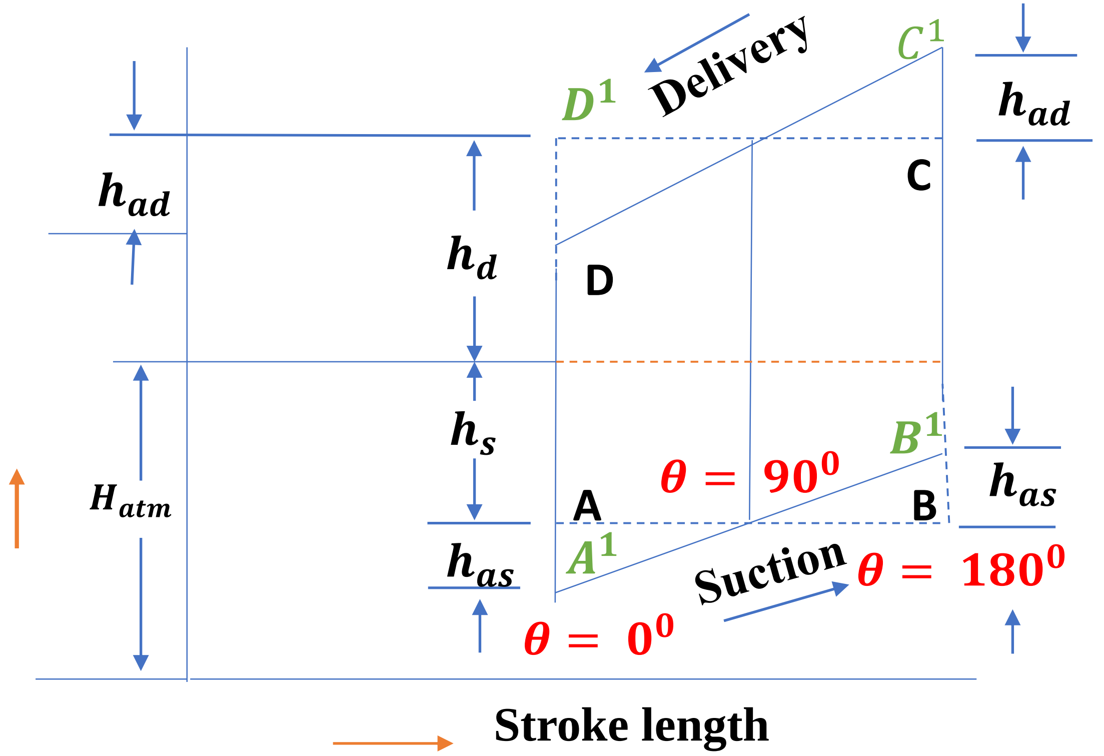
At the end of the delivery stroke, h_d is negative and hence pressure in the cylinder will be (h_d — h_{ad}) above the atmospheric pressure head.

Thus the indicator diagram for delivery stroke is represented by the line C'HD'. Also,

the area of CC'H = Area of DD'H.

From Figure , it is now clear that due to acceleration in suction and delivery pipe, the indicator diagram has changed from **ABCD** to **A’B'C’D’.**

But the area of indicator diagram ABCD = Area A’B'C’D'.



Work done by pump $\propto$ Area of indicator diagram.

work done by pump is proportional to the area of indicator diagram. Hence the work done by the pump on the water remains same.