UNIT -2

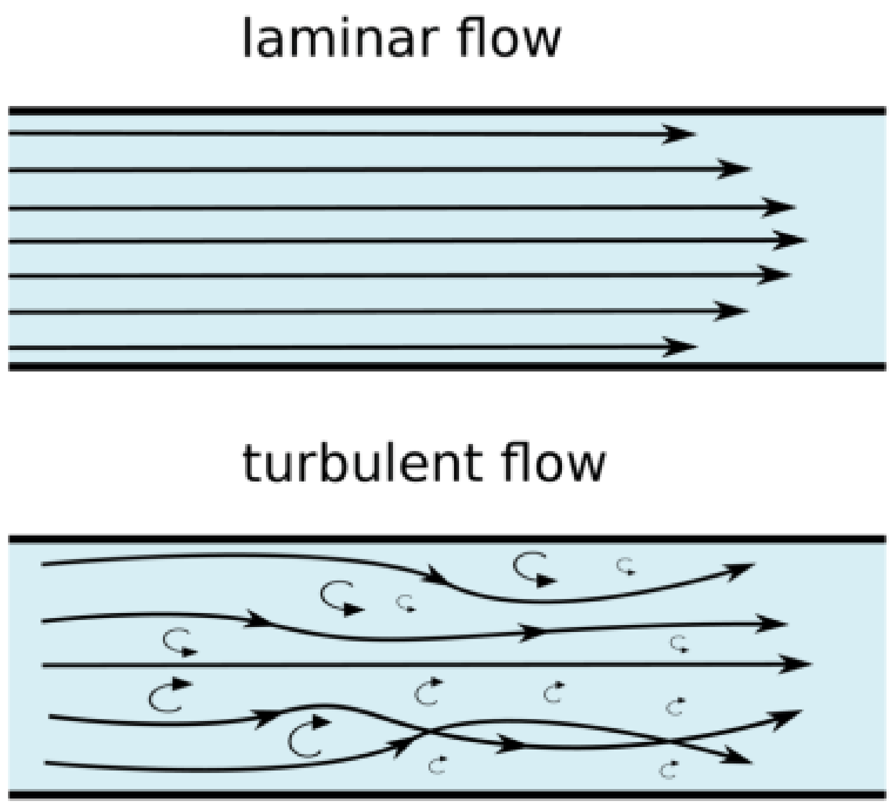
Laminar flow, Turbulent flow & Flow through Pipes

Laminar flow, Turbulent flow: Introduction-Laminar flow – Reynolds experiment; Navier- Stokes equations of motion (Explanation only, No derivation); Relationship between Shear Stress and Pressure Gradient; Flow of viscous fluid in circular pipes-Hagen Poiseuille Law; Flow of viscous fluid between two parallel plates - one plate moving and other at rest ; both plates at rest ; Turbulent flow - Introduction- Loss of head due to friction in pipe flow-Darcy equation- characteristics of turbulent Flow.

Flow through Pipes: Introduction- Major Energy Losses- Darcy-weisbach formula; Minor Energy Losses- Losses due to sudden enlargement, sudden contraction, loss of head at the entrance to pipe, exit of a pipe, loss of head due to bend in pipe and various pipe fittings pipes in series; equivalent pipe; .pipes in parallel; Power Transmission through pipes.

Topic :Turbulent flow

Laminar flow: A laminar flow is one in which paths taken by the individual particles do not cross one another and move along well defined paths . This type of flow is also called stream-line flow or viscous flow.



Examples.

(i) Flow through a capillary tube.

(ii) Flow of blood in veins and arteries.

(iii) Ground water flow.

Turbulent flow: Turbulent flow, type of fluid (gas or liquid) flow in which the fluid undergoes irregular fluctuations, or mixing, in contrast to laminar flow, in which the fluid moves in smooth paths or layers. In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction

Topic :Turbulent flow

Reynolds Experiment :

The type of flow is determined from the Reynolds number i.e., This was demonstrated by O. Reynold in 1883. His apparatus is shown in Figure.

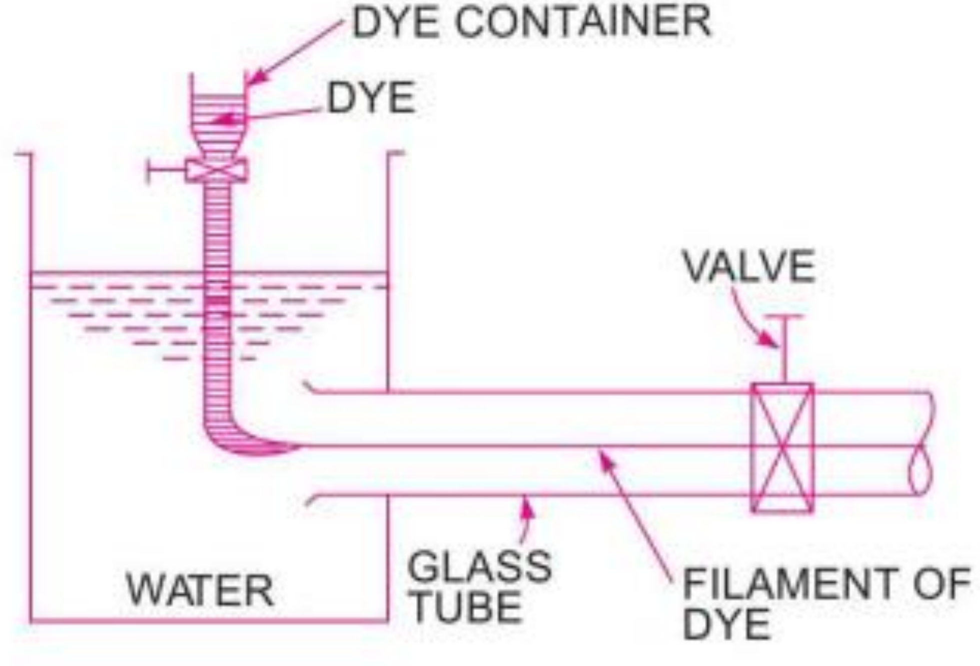
The apparatus consists of:

(i) A tank containing water at constant head,

(ii) A small tank containing some dye,

(iii) A glass tube having a bell-mouthed entrance at one end and a regulating value at other ends.

- The water from the tank was allowed to flow through the glass tube.
- The velocity of flow was varied by the regulating valve.
- A liquid dye having same specific weight as water was introduced into the glass tube as shown in Figure

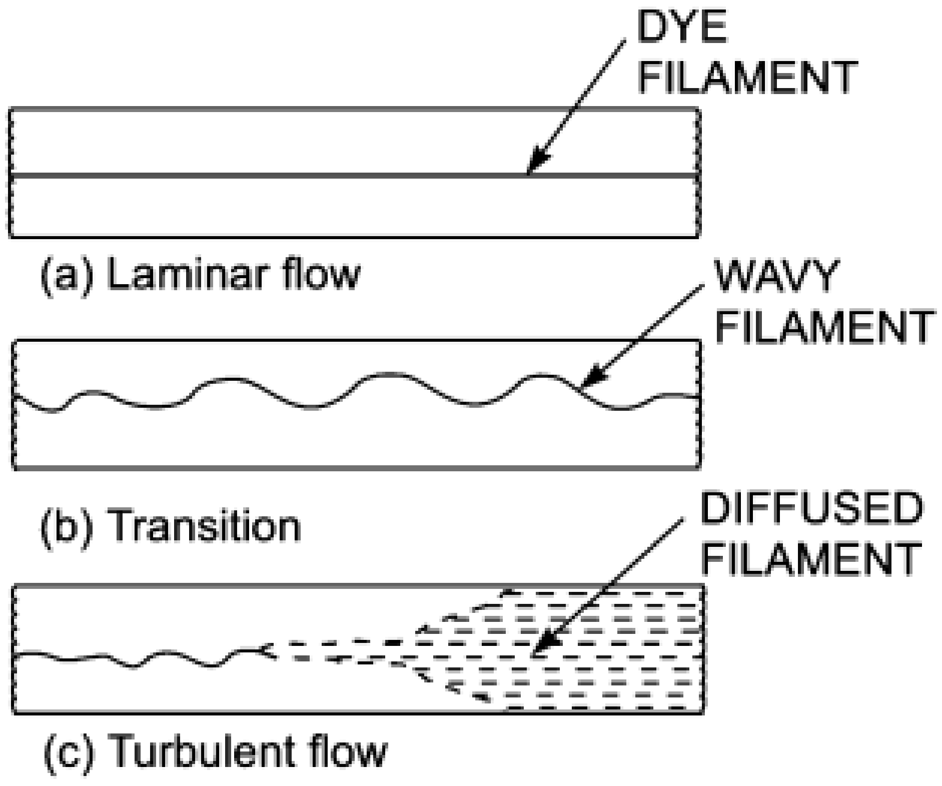


Reynold apparatus.

The following observations were made by Reynold:

(i) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube, which was the case of laminar flow as shown in Figure

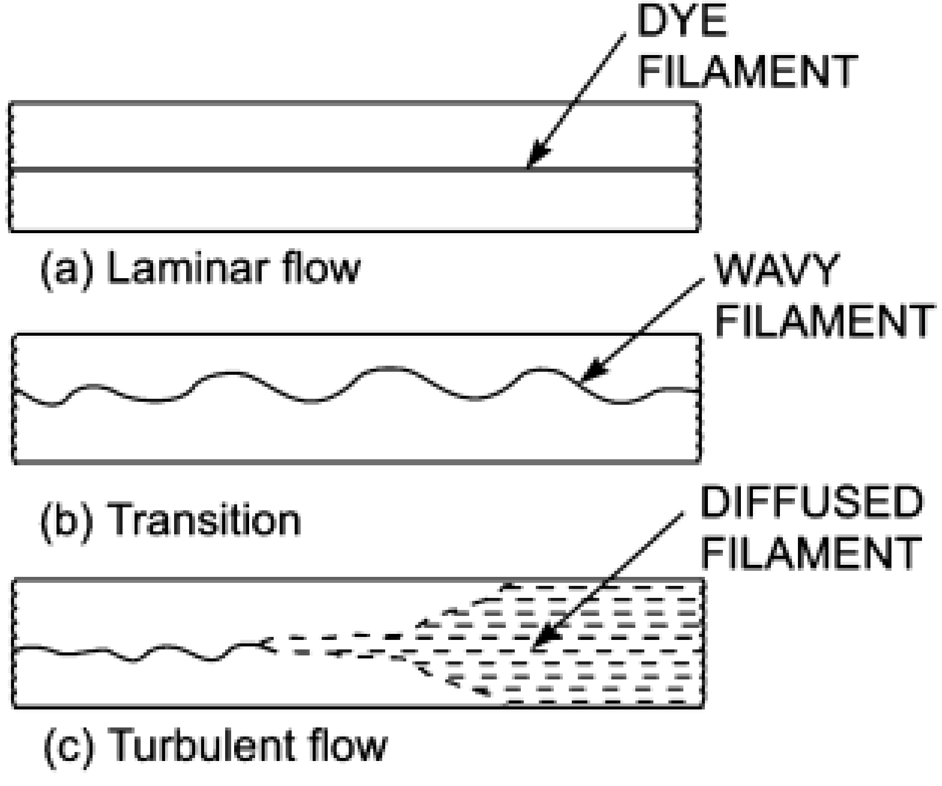
(ii) With the increase of velocity of flow, the dye- filament was no longer a straight-line but it became a wavy one as shown in Figure (b). This shows that flow is no longer laminar.



Different stages of filament.

(iii) With further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water as shown in Figure (c). This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulent flow. Thus in case of turbulent flow the mixing of dye-filament and water is intense and flow is irregular, random and disorderly.

In case of laminar flow, the loss of pressure head was found to be proportional to the velocity but in case of turbulent flow, Reynold observed that loss of head is approximately proportional to the square of velocity.



Different stages of filament.

More exactly the loss of head, h, ∞ V", where n varies from 1.75 to 2.0

FLOW OF VISCOUS FLUID IN CIRCULAR PIPES-HAGEN POISEUILLE LAW

Hagen-Poiseuille theory is based on the following assumptions:

1. The fluid follows Newton's law of viscosity.

2. There is no slip of fluid particles at the boundary (i.e. the fluid particles adjacent to the pipe will have zero velocity).

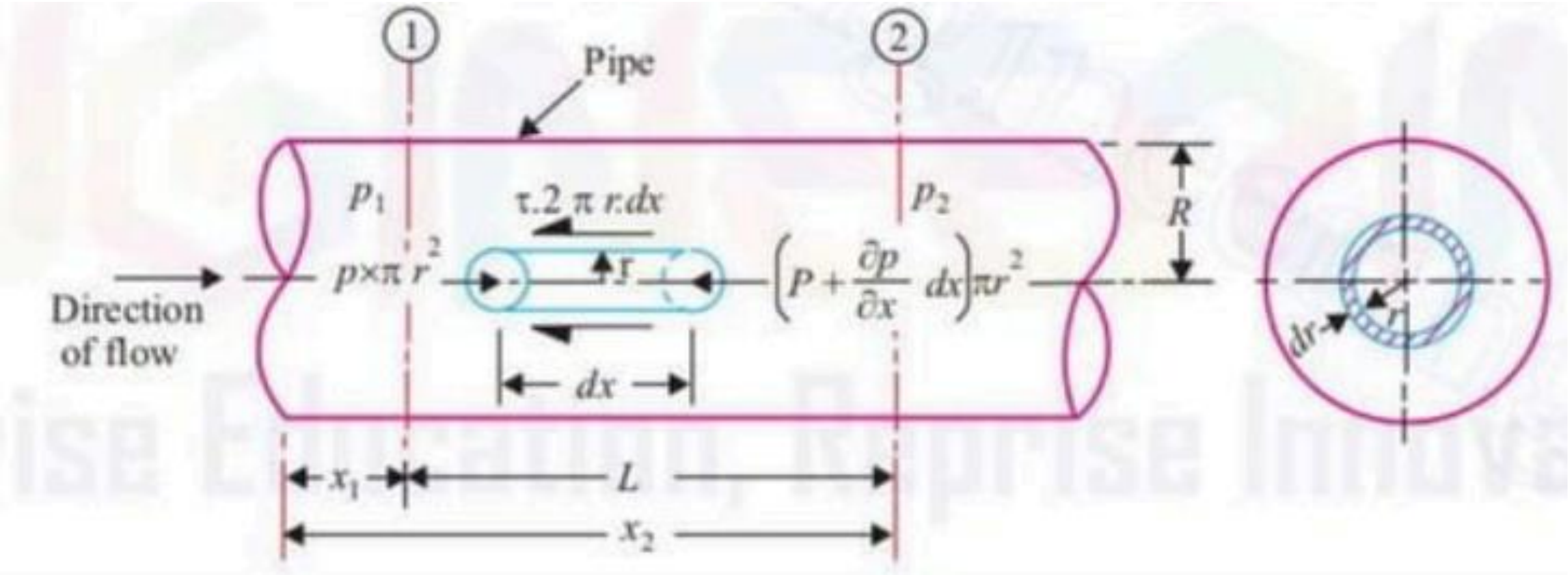
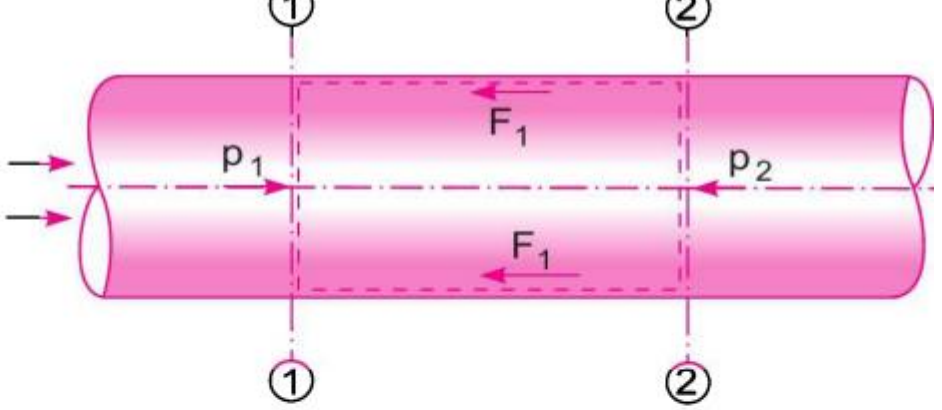


Figure shows a horizontal circular pipe of radius R, having laminar flow of fluid through it.

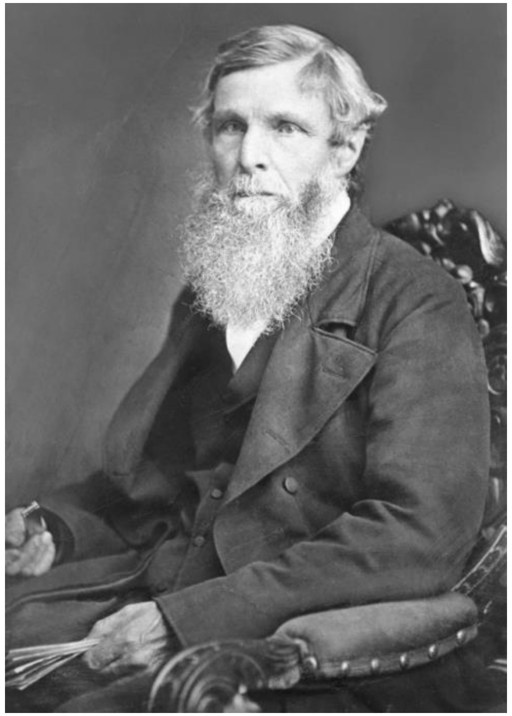
FRICTIONAL LOSS IN PIPE FLOW

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.



Uniform horizontal pipe.

On the basis of his experiments, William Froude gave the following laws of fluid fraction for turbulent flow.



EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F, acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a, in the x-direction. Thus mathematically,

$$F_X = M\, a_x$$

In the fluid flow, the following forces are present:

(i) F_g, gravity force.

(ii) F_p the pressure force.

(iii) F_v force due to viscosity.

(iv) F_t force due to turbulence.

(v) F_c, force due to compressibility.

The net force

Net Force = $F_g + F_p + F_v + F_t + F_c$

the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

Note :Navier –Stokes equation is fundamental equation for Laminar flow

Application for Naiver –Stroke Equation

1) Viscous flow through the circular pipe (Haigen –Poiseuilles Law)
2) Viscous flow between two parallel plates stationary (Plane - Poiseuilles Law)
3) Viscous flow between two parallel plates one plate is stationary (couette flow)

1.FLOW OF VISCOUS FLUID IN CIRCULAR PIPES-HAGEN POISEUILLE LAW

Hagen-Poiseuille theory is based on the following assumptions:

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2. There is no slip of fluid particles at the boundary (i.e. the fluid particles adjacent to the pipe will have zero velocity).

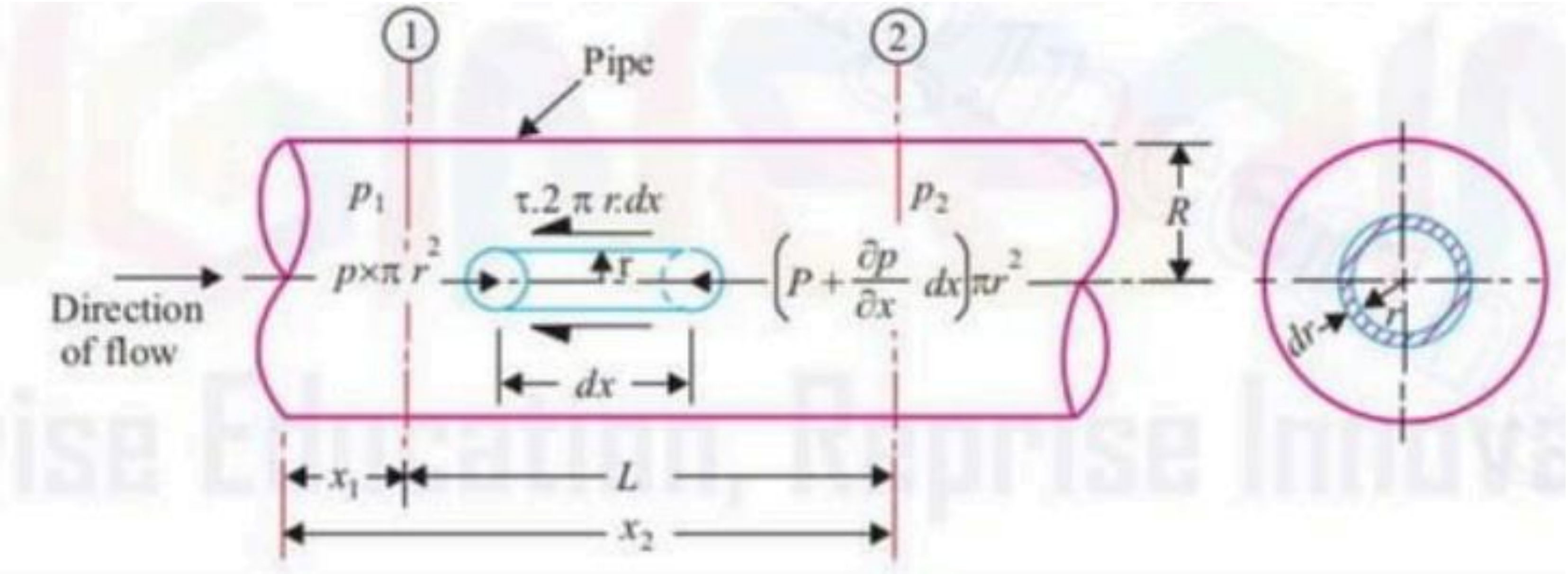


Figure shows a horizontal circular pipe of radius R, having laminar flow of fluid through it.

1.FLOW OF VISCOUS FLUID IN CIRCULAR PIPES-HAGEN POISEUILLE LAW

Assumptions :

1) Steady

2) Flow is fully developed (Velocity is not changing with respect to Length)

(Pressure is not changing with respect to Length)

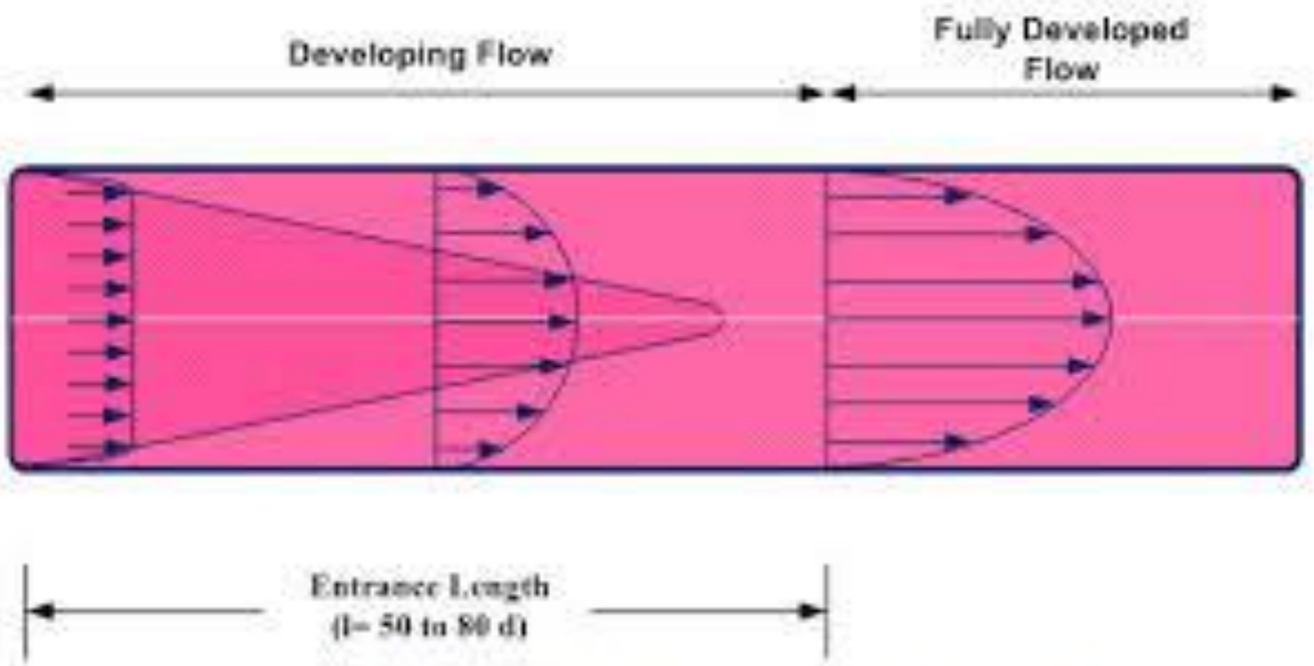
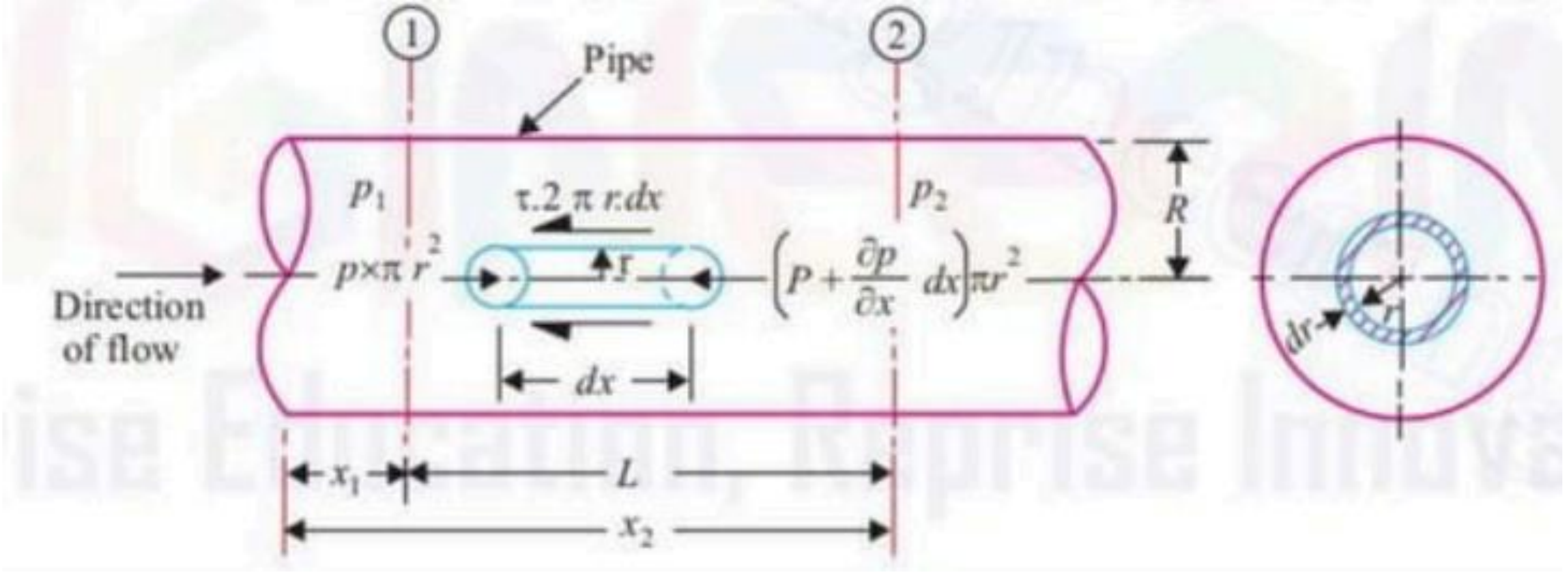


Figure 2 Development of Boundary Layer in Pipe

Shear stress distribution



F = Mass × acceleration

F = M × a

The pressure force, $p \times \pi r^2$

The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$

The shear force, $\tau \times 2\pi r \Delta x$

Figure shows a horizontal circular pipe of radius R, having laminar flow of fluid through it.

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

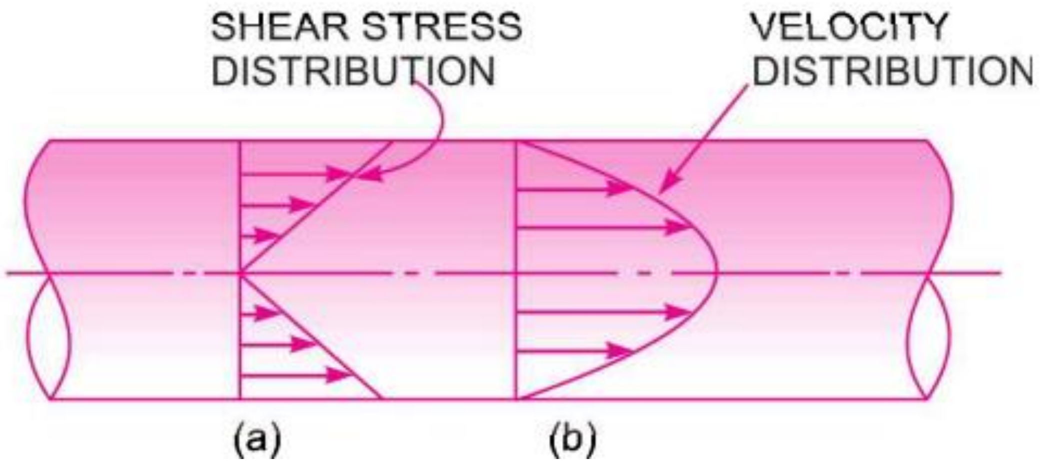
$$-\frac{\partial p}{\partial x} . r - 2\tau = 0$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

Shear stress distribution and velocity distribution

(i) Velocity Distribution. To obtain the velocity distribution across a section, the value of shear stress

$$\tau = \mu \frac{du}{dy}$$



Shear stress and velocity distribution across a section.

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r$$

$$dy = -dr$$

$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x}\, r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\, r^2 + C$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C$$

where C is the constant of integration and its value is obtained from the boundary condition that at r = R, u = 0.

$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C \qquad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$\boxed{u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C}$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \qquad = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

Velocity U = $-\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$

(ii) Ratio of Maximum Velocity to Average Velocity. The velocity is maximum, when **r = 0** in the above equation Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

Discharge (Q)

dQ = velocity at a radius $r \times$ area of ring element

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

Velocity U = $-\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4}$$

$$= \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

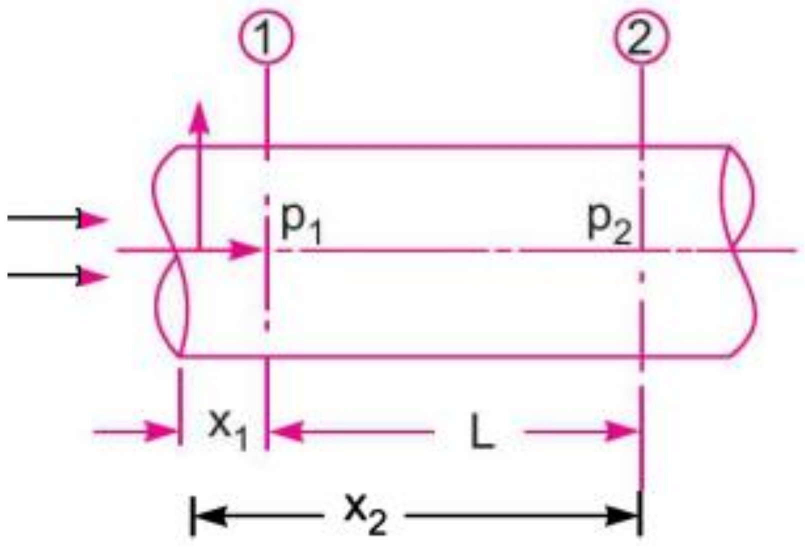
Average velocity, $\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$$

$$\frac{U_{\max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = 2.0$$

(iii)Drop of pressure for a given Length (L)

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \qquad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$



Integrating the above equation w.r.t. x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$$

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2]$$

$$= \frac{8\mu\bar{u}}{R^2} L$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2}$$

$$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$

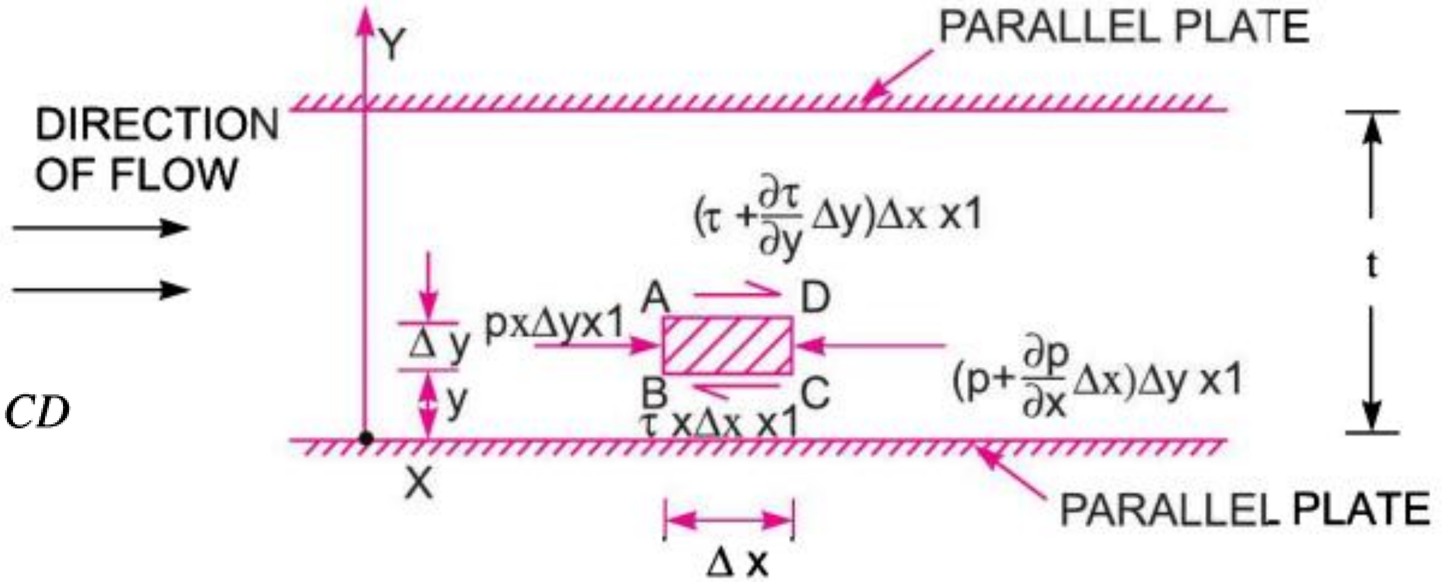
where $p_1 - p_2$ is the drop of pressure.

Loss of pressure head $= \dfrac{p_1 - p_2}{\rho g}$

$$\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$$ is called **Hagen Poiseuille Formula.**

FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES

In this case also, the shear stress distribution, the velocity distribution across a section; the ratio of maximum velocity to average velocity and difference of pressure head for a given length of parallel plates, are to be calculated.



Viscous flow between two parallel plates.

1. The pressure force, $p \times \Delta y \times 1$ on face AB.

2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1$ on face CD

3. The shear force, $\tau \times \Delta x \times 1$ on face BC.

4. The shear force, $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1$ on face AD.

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero.

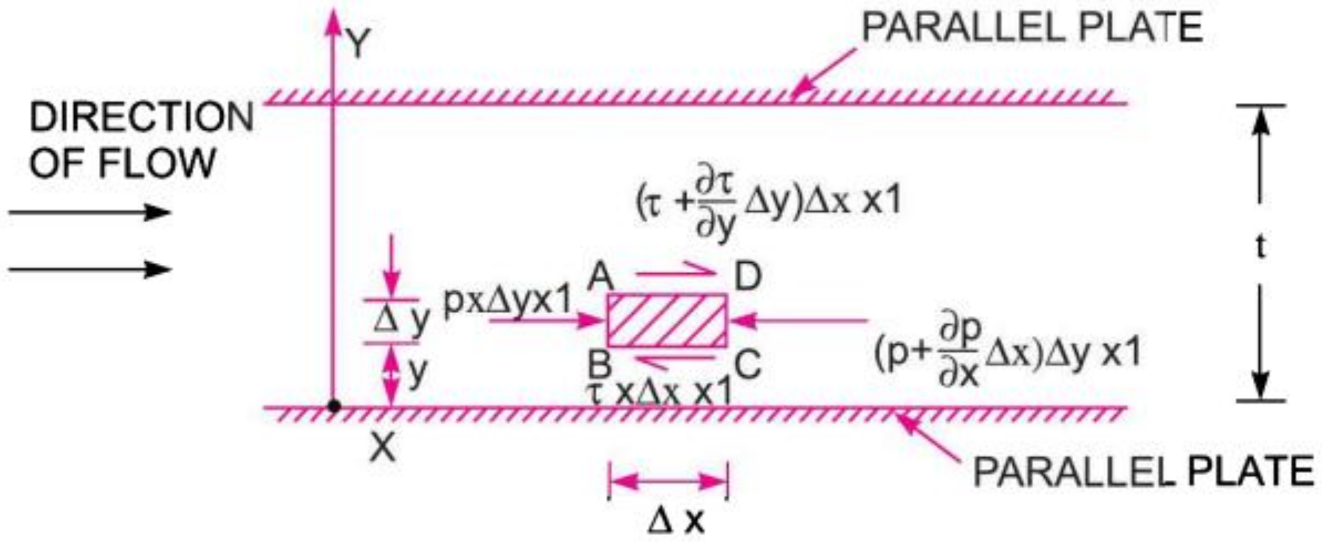
$$p\Delta y \times 1 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1$$

$$-\tau\Delta x \times 1 + \left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1 = 0$$

$$-\frac{\partial p}{\partial x}\Delta x\Delta y + \frac{\partial \tau}{\partial x}\Delta y\Delta x = 0$$

Dividing by $\Delta x\Delta y$, we get $-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = 0$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$



Viscous flow between two parallel plates.

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

(i) Velocity Distribution. To obtain the velocity distribution across a section, the value of shear stress = $\tau = \mu \frac{du}{dy}$ from Newton's law of viscosity for laminar flow is substituted in equation

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) = \mu \frac{\partial^2 u}{dy^2}$$

$$\frac{\partial^2 u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial^2 u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating the above equation w.r.t. y, we get $\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1$

Integrating again $u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$

where C1 and C2 are constants of integration. Their values are obtained from the two boundary conditions that is (i) at y = 0, u = 0 (ii) at y = t, u = 0

$y = 0, u = 0$ in equation (9.8) gives

$0 = 0 + C_1 \times 0 + C_2$ or $C_2 = 0$

$y = t, u = 0$ in equation (9.8) gives

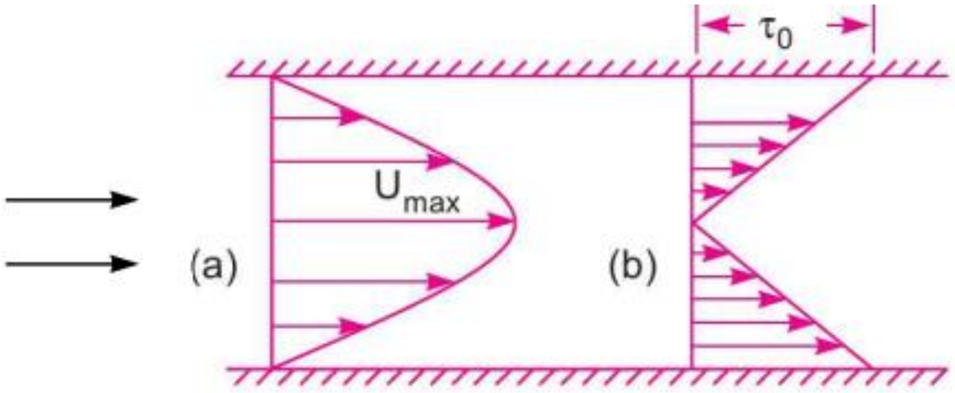
$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2} + C_1 \times t + 0$$

$$C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2 \times t} = -\frac{1}{2\mu}\frac{\partial p}{\partial x} t$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + y\left(-\frac{1}{2\mu}\frac{\partial p}{\partial x} t\right)$$

$$\boxed{u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]}$$



Velocity distribution and shear stress distribution across a section of parallel plates.

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $y = t/2$. Substituting this value in equation (9.9), we get

$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^2}{4}$$

$$= -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2$$

Discharge (Q)

The average velocity, u, is obtained by dividing the discharge (Q) across the section by the area of the section (t x 1). And the discharge Q is obtained by considering the rate of flow of fluid through the strip of thickness **dy** and integrating it. The rate of flow through strip is

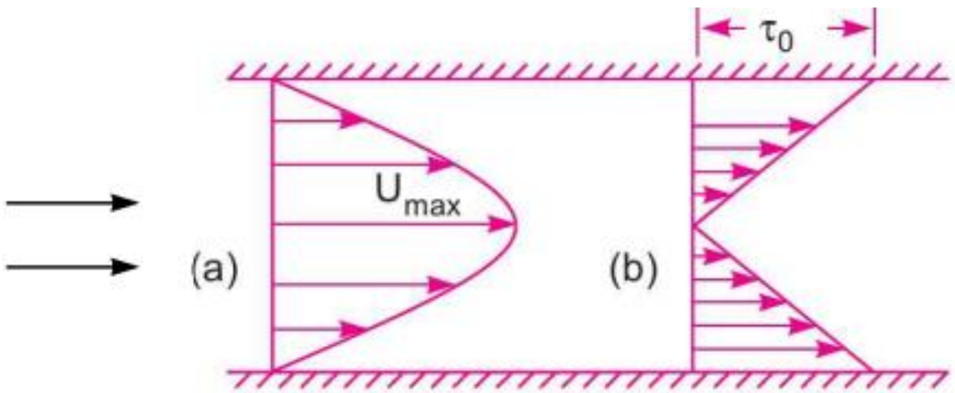
$$dQ = \text{Velocity at a distance } y \times \text{Area of strip}$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]\, dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{ty^2}{2} - \frac{y^3}{3}\right]_0^t \quad = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

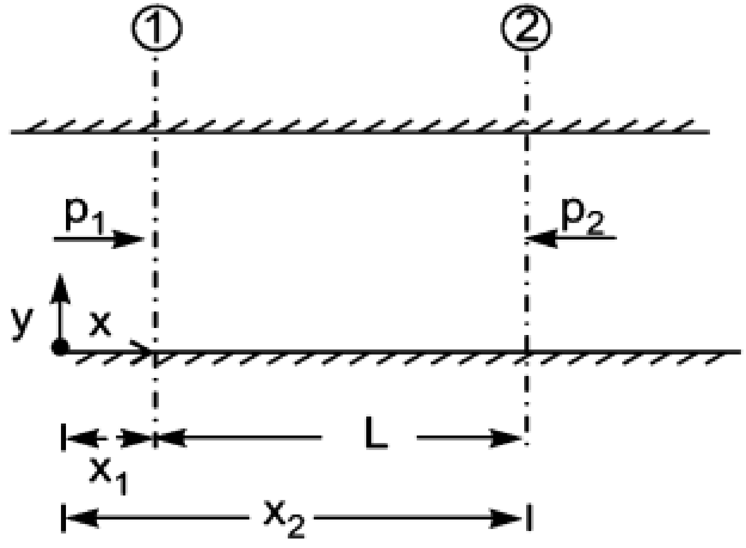
$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^3}{6} \quad = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$



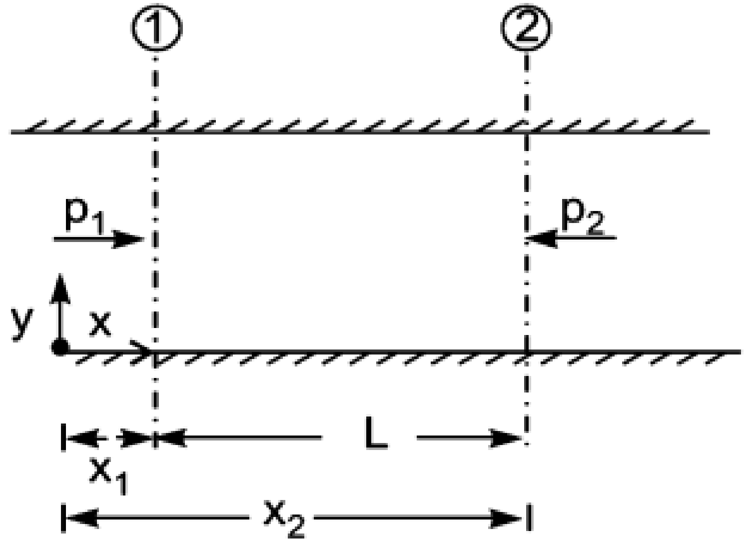
Velocity distribution and shear stress distribution across a section of parallel plates.

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

$$\bar{u} = \frac{Q}{\text{Area}} = -\frac{\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^3}{t \times 1} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$



(*iii*) **Drop of Pressure head for a given Length.**



$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \qquad \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

Integrating this equation w.r.t. x, we get

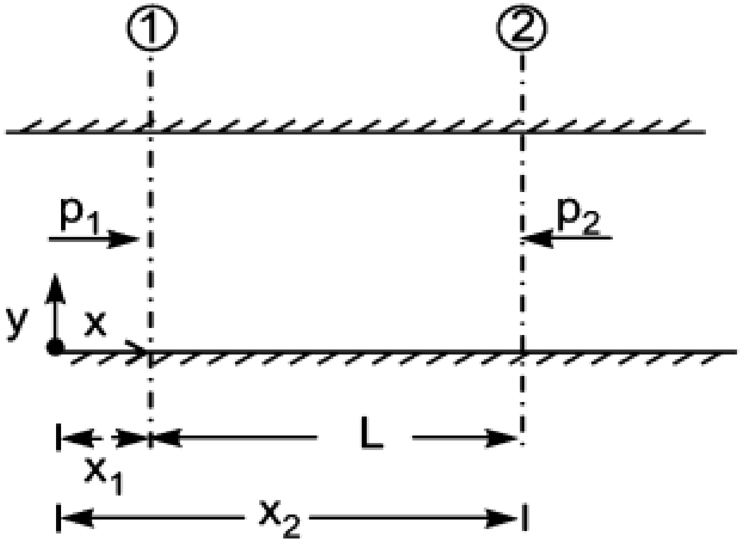
$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}\,dx$$

$$p_1 - p_2 = -\frac{12\mu\bar{u}}{t^2}[x_1 - x_2] \quad = \frac{12\mu\bar{u}}{t^2}[x_2 - x_1]$$

$$p_1 - p_2 = \frac{12\mu\bar{u}L}{t^2} \qquad [\because\ x_1 - x_2 = L]$$

If h_f is the drop of pressure head, then

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{12\mu \bar{u} L}{\rho g t^2}$$

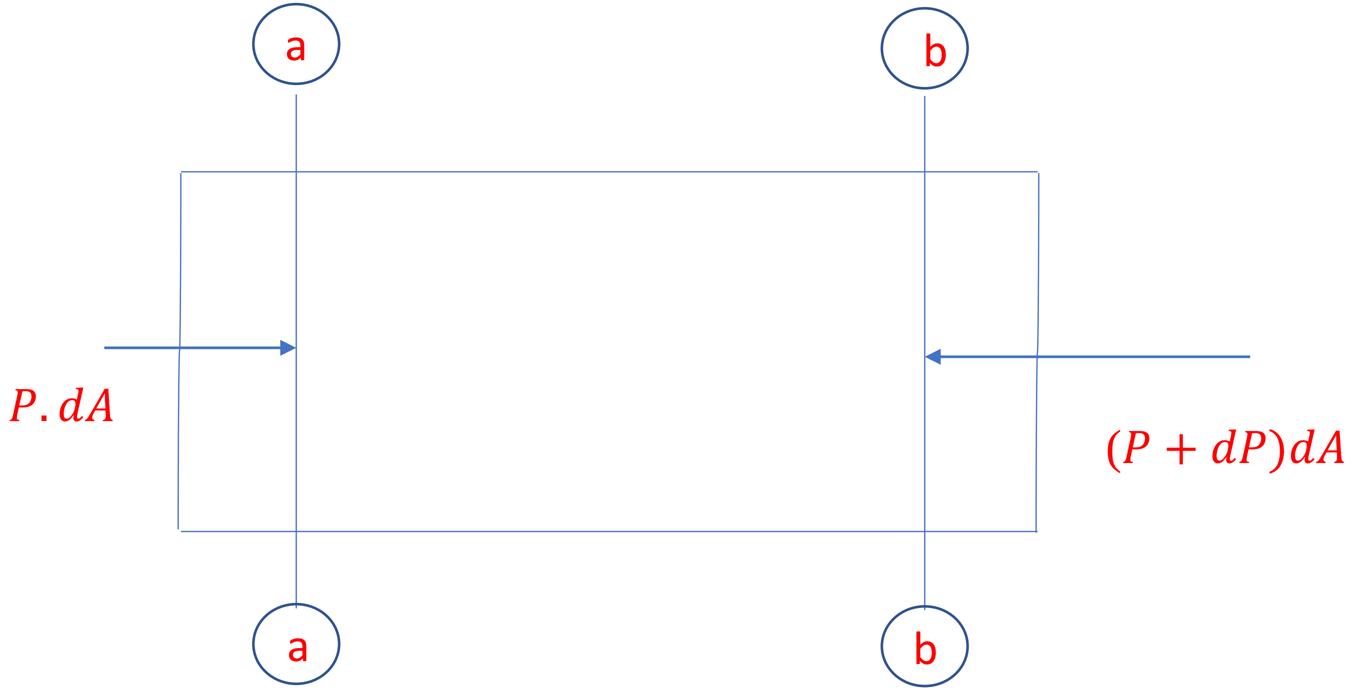


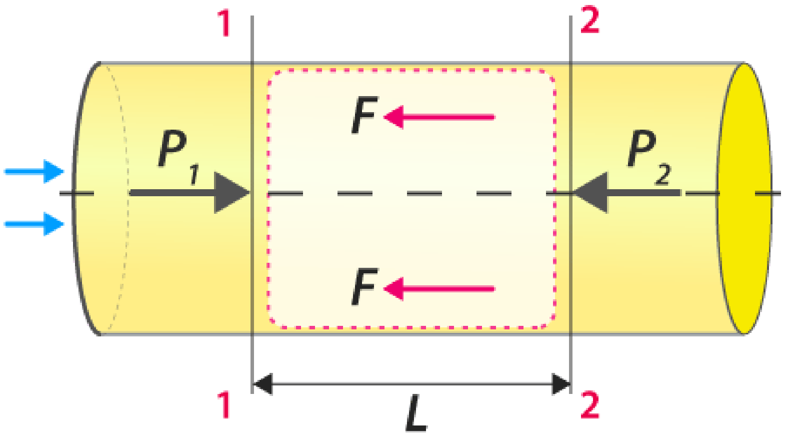
(*iv*) Shear Stress Distribution.

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(ty - y^2\right)\right] = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\boxed{\tau = -\frac{1}{2}\frac{\partial p}{\partial x}\,[\,t - 2y]}$$





Assumptions :

Frictional resistance :

1)Varies as V^n (n= 1.5 to 2)

V = velocity

2)Proportional density of fluid

3)Proportional to area of surface in contact

4)Independent of pressure

5)Depend on nature of surface contact.

Derivation

$A_1 V_1 = A_2 V_2$
(Continuity Equation)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$ (Ideal Fluid)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$ (Real Fluid)

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h_L \quad \ldots\ldots\ldots\ldots (1)$$

f^1 = Frictional resistance per unit wetted area per unit velocity

Frictional Resistance = $F_1 = f^1 \pi D L V^n$

$F_1 = f^1 \pi D L V^2$ (n=2)

Topic :Turbulent flow

Flow as in x-direction

Local acceleration $a_c = 0$

As per Newtons 2nd law: F = ma

F (Resultant force) = 0

$$P_1 A - P_2 A - F_1 = 0$$

$$P_1 - P_2 = \frac{F_1}{A}$$

$$P_1 - P_2 = \frac{f^1 \pi D L V^2}{\frac{\pi}{4} D^2}$$

$$P_1 - P_2 = \frac{f^1 4 L V^2}{D}$$

(For horizontal pipe)

$$\frac{P_1 - P_2}{\rho g} = \frac{f^1 4 L V^2}{\rho g D}$$

(For inclined pipe)

$$\frac{f^1}{\rho} = \frac{C_f}{2}$$

$$h_f = \frac{C_f 4 L V^2}{2.g.D}$$

$4 C_f = f$ $\quad f = Friction\ factor\ or\ Darcy\ friction\ factor$

$$h_f = \frac{f L V^2}{2.g.D}$$

Darcy Wheishbach equation

Important Point:

$$h_f = \frac{f\,L\,V^2}{2.g.D}$$ valid for Laminar and Turbulent flow

$$h_f = \frac{32\,\mu\,V\,L}{\rho\,g\,D}$$ As per Haigen Poiseuille's equation

$$\frac{f\,L\,V^2}{2.\,g.\,D} = \frac{32\,\mu\,V\,L}{\rho\,g\,D}$$

$$f = \frac{64\,\mu}{\rho\,V\,D}$$

$$f = \frac{64}{R_e}$$ $R_e = Reynolds\ number\ for\ pipe\ for\ laminar\ flow$

$$\boldsymbol{C_f = \frac{f}{4} = \frac{64}{R_e} = \frac{16}{R_e}}$$

Topic :Turbulent flow

$$h_f = \frac{f\,L\,V^2}{2.g.D}$$

(Darcy weisbatch equation) valid for Laminar and Turbulent flow

$$h_f = \frac{32\,\mu\,V\,L}{\rho\,g\,D}$$

(As per Haigen Poiseuille's equation (valid for fully developed laminar flow)

$$h_f = \frac{32\,\mu\,V\,L}{\rho\,g\,D}$$

Co efficient of friction in terms of wall shear stress

F= Resistance force in the pipe

τ_0 = Shear stress on the wall of the pipe

$$P_1A - P_2A - F = 0$$

$$(P_1\text{-}P_2)\,A = F$$

Frictional resistance in the pipe

$$F = \tau_0 .2\pi . R . L$$

$$F = \tau_0 . \pi . D . L$$

$$(P_1\text{-}P_2)\,A = \tau_0 . \pi . D . L$$

$$(P_1\text{-}P_2) = \frac{\tau_0 .\pi.D.L}{A}$$

$$(P_1\text{-}P_2) = \frac{\tau_0 .\pi.D.L}{\frac{\pi}{4} D^2}$$

$$\boxed{(P_1\text{-}P_2) = \frac{4.\tau_0.L}{D}}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{4\, C_f\, L\, V^2}{2.g.D}$$

(For Horizontal pipe)

$$P_1\text{-}P_2 = \frac{4\, C_f\, L\, V^2}{2.g.D} \times \frac{\rho g}{}$$

(For inclined pipe)

$$\frac{4\, C_f\, L\, V^2}{2.g.D} \times \frac{\rho g}{} = \frac{4.\tau_0.L}{D}$$

$$C_f = \frac{2.\tau_0.L}{\rho V^2}$$

Co-efficient of friction in terms of wall shear stress

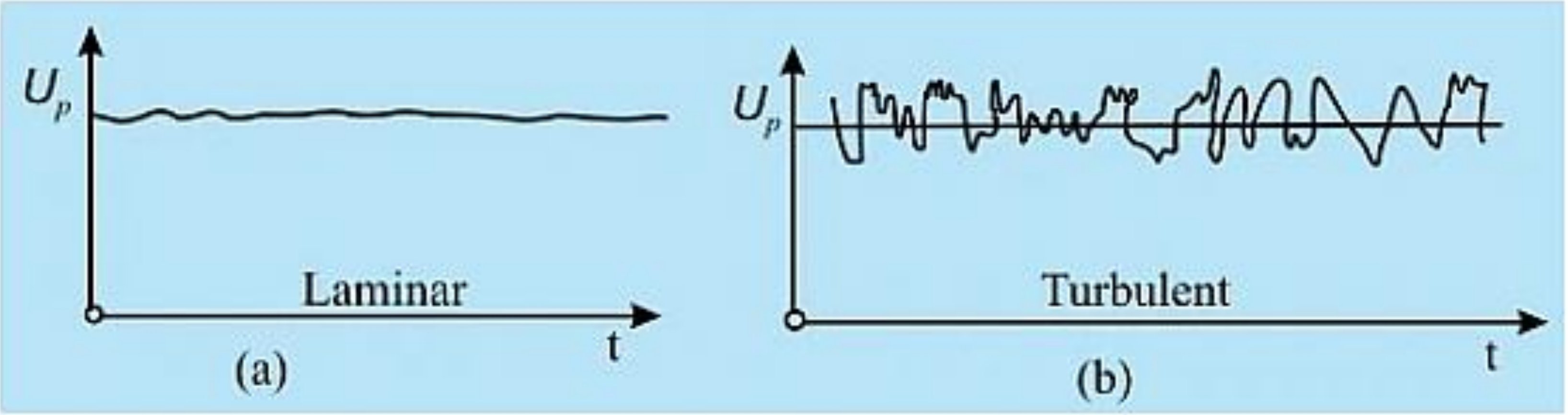
Darcy friction factor:

$$f = 4.C_f$$

$$f = \frac{4.\tau_0.L}{\rho V^2}$$

v= Mean velocity

characteristics of turbulent Flow.



u= f(x,y,z)	u= f(x,y,z,t)
v= f(x,y,z)	v= f(x,y,z,t)
w= f(x,y,z)	w= f(x,y,z,t)

Reynolds experiment for turbulent

Shear stress : we initially used Newtons law of viscosity that for steady and laminar flow.

But in the case of turbulent flow

Mean velocity component + fluctuation velocity component

U = Mean velocity component ($\bar{u}$)+ fluctuation velocity component (u^1)

$$U = (\bar{u}) + (u^1)$$

$$v = (\bar{v}) + (v^1)$$

$$w = (\bar{w}) + (w^1)$$

Reynolds experiment for turbulent

We initially used newtons law of viscosity that was for steady and laminar flow

But in case of turbulent flow mean velocity component and fluctuation velocity component exist

Shear stress

Reynolds given = $\boldsymbol{\tau = \rho u^1 v^1}$

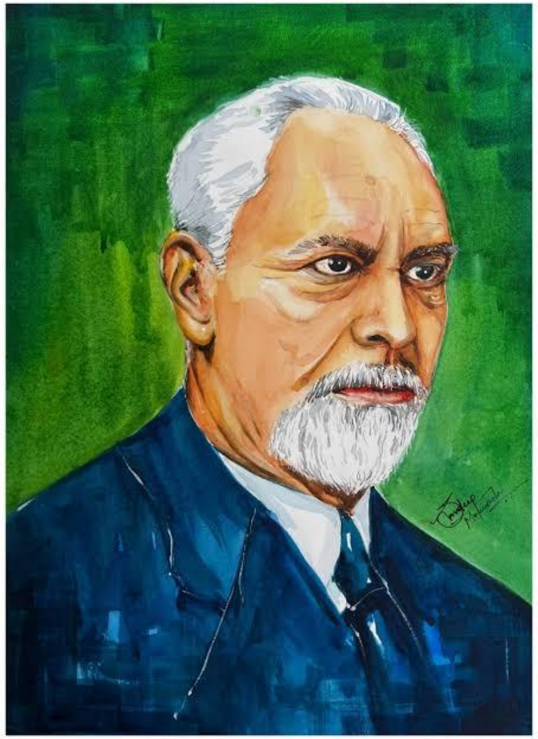
u^1 = In X-direction fluctuating component

v^1 = In Y-direction fluctuating component

Average shear stress observed in particular range of time :

Time average shear stress ($\bar{\tau} = \rho u^1 v^1$)

Prandtl mixing length (L) :



- Mixing length not flow along the length
- Particle jump one plane to another plane. Fluid particle jump moment is always conserved
- It is distance travelled by lump of particles such that moment of fluid particles does not change

Mixing length (L)

Prandtl given based on the experiment

$$u^1 = L\left(\frac{du}{dy}\right)$$

$$v^1 = L\left(\frac{du}{dy}\right)$$

$$\bar{\tau} = \rho L^2 \left(\frac{du}{dy}\right)^2$$

Value of shear stress in turbulent flow in terms of mixing length

Turbulent flow:

Velocity of the fluid particles near the wall is very less as compared to for distance of fluid particles from the wall even it is turbulent flow fluid particles velocity is very less due to attraction of forces offered by solid boundary.

The layer inside which the flow is laminar in turbulent flow field such layer is known as laminar sub layer.

Total sear stress = shear stress near the wall + shear stress due to turbulence

$$\tau = \mu \frac{du}{dy} + \rho L^2 \left(\frac{du}{dy}\right)^2$$

Prandtl mixing length (L) :

L = K.y k = Karman's constant = 0.4

Mean shear stress = $\bar{\tau} = \rho L^2 \left(\frac{du}{dy}\right)^2$

$$\bar{\tau} = \rho (K.y)^2 \left(\frac{du}{dy}\right)^2$$

$$\left(\frac{du}{dy}\right)^2 = \frac{\bar{\tau}}{\rho (K.y)^2}$$

$$\left(\frac{du}{du}\right) = \sqrt{\frac{\bar{\tau}}{\rho (K.y)^2}} = \frac{1}{K.y}\sqrt{\frac{\bar{\tau}}{\rho}}$$

Topic :Turbulent flow

$$\left(\frac{du}{du}\right) = \sqrt{\frac{\bar{\tau}}{\rho(K.y)^2}} = \frac{1}{K.y}\sqrt{\frac{\bar{\tau}}{\rho}}$$

Assume if y is so small $\tau = \tau_0$

$$\left(\frac{du}{du}\right) = \frac{1}{K.y}\sqrt{\frac{\tau_0}{\rho}}$$

Shear velocity = $u^* = \sqrt{\frac{\tau_0}{\rho}}$

$$\left(\frac{du}{du}\right) = \frac{1}{K.y}u^*$$

$$\tau_0 = \frac{Force}{area} = \frac{M.L.T^2}{L^2} = M.L^{-1}T^{-2}$$

$$\rho = \text{Density} = \frac{Mass}{Volume} = \frac{kg}{L^3} = \frac{M}{L^3} = M.L^{-3}$$

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{M.L^{-1}T^{-2}}{M.L^{-3}}} = \frac{L}{T} \text{ meter/sec}$$

Velocity distribution for turbulent flow

$$\int du = \int \frac{1}{K.y} u^* \, dy$$

$$u = \frac{1}{k} u^* \ln y + c$$

c = constant of integration

Boundary conditions : $U = U_{max}$ at y=R

$$U_{max} = \frac{1}{k} u^* \ln R + c$$

$$C = U_{max} - \frac{1}{k} u^* \ln R$$

$$U = \frac{u^*}{k} \ln y + U_{max} - \frac{1}{k} u^* \ln R$$

$$U = U_{max} + \frac{u^*}{k} (\ln y - \ln R)$$

$$U = U_{max} + \frac{u^*}{k} \ln\left(\frac{y}{R}\right)$$

K = 0.4

$$U = U_{max} + \frac{u^*}{0.4} \ln\left(\frac{y}{R}\right)$$

Velocity distribution for a turbulent flow in the pipe

$$U = U_{max} + 2.5\, u^* \ln\left(\frac{y}{R}\right)$$

Velocity distribution for a turbulent flow in the pipe

$$U = U_{max} + 2.5\, u^{*} \ln\left(\frac{y}{R}\right)$$

Velocity distribution for a turbulent flow in the pipe

Velocity distribution is logarithmic.

Prandtl universal velocity distribution for turbulent flow

- Valid For smooth as well as rough pipe.

(Any surface is perfectly smooth)

Laminar sub Layer

Ɛ = Average surface roughness

ρ^1 = thickness of laminar sub layer

$\frac{\varepsilon}{\rho^1} < 0.25$ pipe is smooth

$\frac{\varepsilon}{\rho^1} > 6$ pipe is rough surface

$0.25 < \frac{\varepsilon}{\rho^1} < 6$ Transition

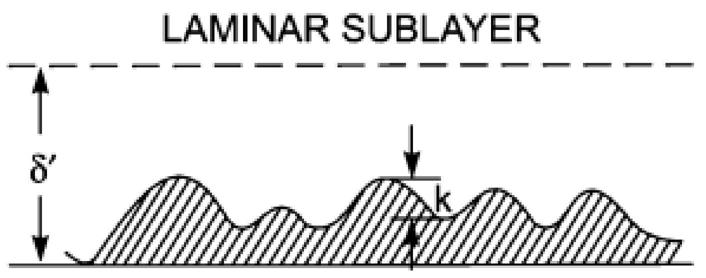
Velocity defect:

$$U = U_{max} + 2.5\, u^* \ln\left(\frac{y}{R}\right)$$

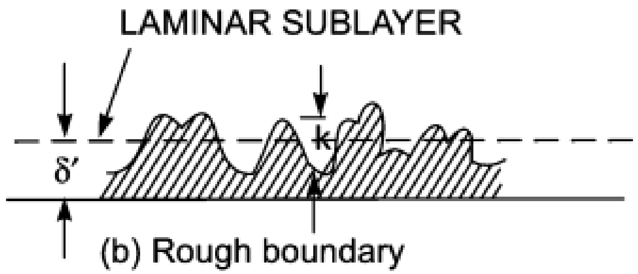
$$U_{max} - U = 2.5\, u^* \ln\left(\frac{R}{y}\right)$$

u at any location with respect to **y**

Hydro dynamically Smooth or rough surface



(a) Smooth boundary



(b) Rough boundary

Reynolds number (R_e Number)

Reynolds number in terms of surface roughness

$$R_e \text{ Number} = \frac{\varepsilon u^*}{\nu} = \frac{\varepsilon u^* \rho}{\mu}$$

$$\frac{\varepsilon u^*}{\nu} < 4 \quad \text{smooth pipe}$$

$$\frac{\varepsilon u^*}{\nu} > 100 \quad \text{rough pipe}$$

$$4 < \frac{\varepsilon u^*}{\nu} < 100 \quad \text{transition}$$

Velocity distribution for turbulent flow in smooth pipe

$$u = \frac{1}{k} u^{*} \ln y + c$$

C=constant

u^{*} = shear velocity

K =karman's constant

y = o at solid boundary

$$u = \frac{1}{k} u^{*} \ln (0) + c$$

Velocity showing negativity infinity but at some point negative infinity convert to positive infinity at that point y= y^{1}

So that at that point u = 0 ; y= y^{1}

$$u = \frac{1}{k} u^* \ln y + c$$

Boundary conditions : y = y^1 ,U = 0

$$0 = \frac{1}{k} u^* \ln y + c$$

$$C = -\frac{1}{k} u^* \ln y^1$$

$$U = \frac{1}{k} u^* \ln y \; - \frac{1}{k} u^* \ln y^1$$

$$\frac{U}{u^*} = 2.5\, u^* \ln \frac{y}{y^1}$$

y^1 is distance of the point measured perpendicular to solid boundary at u = 0

In case of laminar sub layer is parabolic but above laminar sub layer logarithmic.

Nochrodes Experiment : y^1 *value is depond on* ρ^1 on smooth flow pipe

$$y^1 \propto \rho^1$$

$$y^1 = \frac{\rho^1}{107}$$

$$\rho^* = \frac{11.6\, v}{u^*}$$

$$y^1 = \frac{11.6v}{u^* 107}$$

$$y^1 = \frac{0.108v}{u^*}$$

$$\frac{U}{u^*} = 2.5\, u^* \ln \frac{y}{y^1}$$

$$\frac{u}{u^*} = 2.5 \ln \frac{y}{\frac{0.108\, v}{u^*}}$$

$$\frac{u}{u^*} = 2.5\, ln\left(\frac{yu^*}{v}\ 9.259\right)$$

$$\frac{u}{u^*} = 2.5\, ln\left(\frac{yu^*}{v} + 5.56\right)$$

Velocity distribution in rough pipe :

$$\frac{U}{u^*} = 2.5\, u^* \ln \frac{y}{y^1}$$

For rough pipe

Average roughness value is more

So laminar sub layer destroyed

Nikordam's:

$$y^1 \propto \varepsilon^1$$

$$y^1 \propto \varepsilon/30$$ (For rough pipe)

$$\frac{U}{u^*} = 2.5\, u^* \ln \frac{y}{\varepsilon/30}$$

$$\frac{U}{u^*} = 2.5\, u^* \ln \frac{y * 30}{\varepsilon}$$

$$\frac{U}{u^*} = 2.5 \ln \frac{y}{\varepsilon} + 2.5 \ln 30$$

$$\frac{U}{u^*} = 2.5 \ln \frac{y}{\varepsilon} + 8.5$$


FLOW THROUGH
PIPES

LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

1. Major Energy Losses

This is due to friction and it is calculated by the following formulae:

(a) Darcy-Weisbach Formula

(b) Chezy's Formula

2. Minor Energy Losses

This is due to

(a) Sudden expansion of pipe

(b) Sudden contraction of pipe

(c) Bend in pipe

(d) Pipe fittings etc.

(e) An obstruction in pipe.

Flow through pipes

Major Losses (95% of contribution in total loss)

Friction so Darcy weisbach equation

$$h_f = \frac{f\,L\,V^2}{2.g.D}$$

Q = A.V

$$Q = \frac{\pi}{4} D^2 \times V$$

$$V = \frac{4.Q}{\pi D^2}$$

$$h_l = \frac{fl16Q^2}{\pi^2 D^4 gD}$$

$$h_l = \frac{flQ^2}{12D^5}$$

Chezy,s formula :

V= C $\sqrt{mi}$ Empirical formula (No prove but only get by experiment)

V = average velocity

C = Chezy,s constant

m= Hydraulic mean depth

i= Hydraulic slope

Topic : Flow through pipes

$$m = \frac{area}{wetted\ perimeter} = \frac{\pi D^2}{4\,\pi\,D}$$

$$m = \frac{D}{4}$$

$$i = \tan\theta = \frac{h_l}{4}$$

$$v = c\sqrt{\frac{D\,h_l}{4L}}$$

$$h_l = \frac{4LV^2}{C^2\,D}$$

$$\frac{f\,L\,V^2}{2.g.D} = \frac{4LV^2}{C^2\,D}$$

$$C^2 = \frac{8g}{f}$$

$$C = \sqrt{\frac{8g}{f}}$$

Important formulas to solve the problems (Flow through pipes)

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 . f . L . V^2}{d \times 2g} \qquad \text{...(11.1)}$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

Problem 11.1 *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν for water = 0.01 stoke.

Dia. of pipe,	$d = 300$ mm = 0.30 m
Length of pipe,	$L = 50$ m
Velocity of flow,	$V = 3$ m/s
Chezy's constant,	$C = 60$
Kinematic viscosity,	$\nu = 0.01$ stoke = 0.01 cm^2/s = $0.01 \times 10^{-4}\ m^2/s$.

(*i*) **Darcy Formula** $h_f = \dfrac{4 . f . L . V^2}{d \times 2g}$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

$$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$$

$$f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$$

Head lost, $h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81}$ = **.7828 m. Ans.**

(*ii*) **Chezy's Formula.**

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \frac{d}{4} = \frac{0.30}{4} = 0.075$ m

$3 = 60\sqrt{.075 \times i}$ $\qquad i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$

$$i = \frac{h_f}{L} = \frac{h_f}{50}$$

$\frac{h_f}{50} = .0333$ $\qquad h_f = 50 \times .0333$ = **1.665 m. Ans.**

Problem 11.2 *Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.*

Length of pipe, $L = 2000$ m

Discharge, $Q = 200$ litre/s $= 0.2$ m^3/s

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Let the diameter of pipe $= d$

Velocity of flow, $$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $$m = \frac{d}{4}$$

Loss of head per unit length, $$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

$$V = C\sqrt{mi}$$

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times .002}$$

$$d = \sqrt[5]{0.0518} = (.0518)^{1/5}$$ **= 553 mm. Ans.**

Problem 11.3 *A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres per sec. Find the head lost due to friction for a length of 50 m of the pipe.*

Problem 11.4 *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take* ν *= .29 stokes.*

Problem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \dfrac{4 . f . L . V^2}{d \times 2g}$.

Problem 11.6 *Water is flowing through a pipe of diameter 200 mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5 m if the co-efficient of friction is given by* $f = 0.02 + \dfrac{.09}{R_e^{\ 0.3}}$, *where* R_e *is Reynolds number. The kinematic viscosity of water = .01 stoke.*

Problem 11.7 *An oil of sp. gr. 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 litres/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.*

Minor losses : Minor losses constitute of 5% o the total losses and it consider

(a) Sudden expansion loss
(b) Sudden contraction loss
(c) Entry loss
(d) Exit loss
(e) Bending losses

As magnitude of minor losses are small in reference to the frictional losses

a) Sudden expansion loss:

1)steady

2)the pressure in Eddy region is taken as upstream pressure

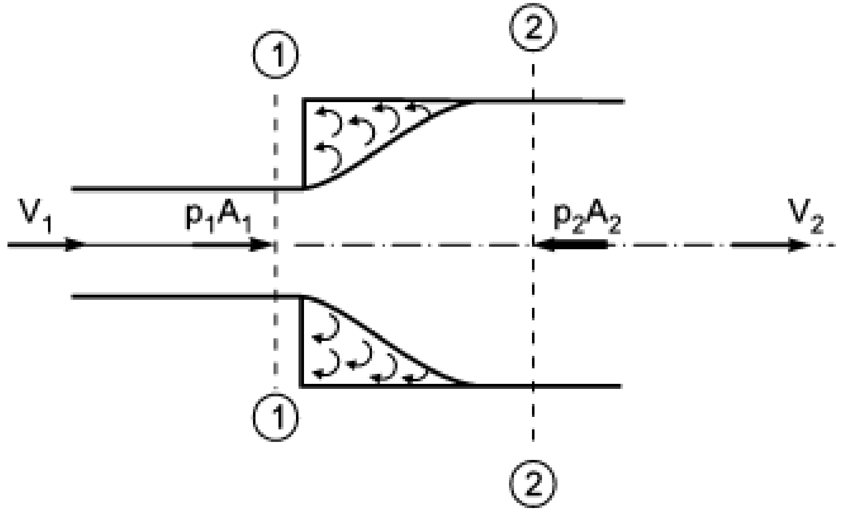


Fig. 11.1 *Sudden enlargement.*

Momentum integral function:

$\sum F$ =ρQ (v-u)

$$P_1A_1 + P_2A_2 - P_1A_1 - P_2A_2 = \rho Q\,(V_2 - V_1)$$

$$P_1A_1 + P_1(A_2 - A_1) - P_2A_2 = \rho Q\,(V_2 - V_1)$$

$$A_2(P_1 - P_2) = \frac{\rho Q\,(V_2 - V_1)}{A_2}$$

$$P_1 - P_2 = \rho\, V_2(V_2 - V_1)$$

$$\frac{P_1 - P_2}{\omega} = \frac{V_2{}^2 - V_2V_1}{g}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + (h_l)_{S.E}$$

$$\frac{P_1 - P_2}{\omega} - \frac{V_2^2}{2g} + \frac{V_1^2}{2g} = (h_l)_{S.E}$$

$$(h_l)_{S.E} = \frac{(V_1^2 - V_2^2)}{2g}$$

$$(h_l)_{S.E} = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

$$(h_l)_{S.E} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

2)Exit loss

$$A_2 = \infty$$

$$(h_l)_{S.E} = \frac{V^2}{2g}$$

3)Sudden in contraction:

$$c_c = \frac{A_c}{A_2}$$

$$A_1 V_C - A_2 V_2$$

$$\frac{V_C}{V_2} = \frac{A_2}{A_C} = \frac{1}{c_c}$$

$$(h_l)_{S.C} = \frac{(V_C - V_2)^2}{2g}$$

$$(h_l)_{S.C} = \frac{V^2}{2g}\left(\frac{1}{C_C} - 1\right)^2$$

$$(h_l)_{S.C} = \frac{0.5\, V_2^2}{2g}$$

Entry loss :

$$(h_l)_{Sudden\ entry} = \frac{0.5\, V^2}{2g}$$

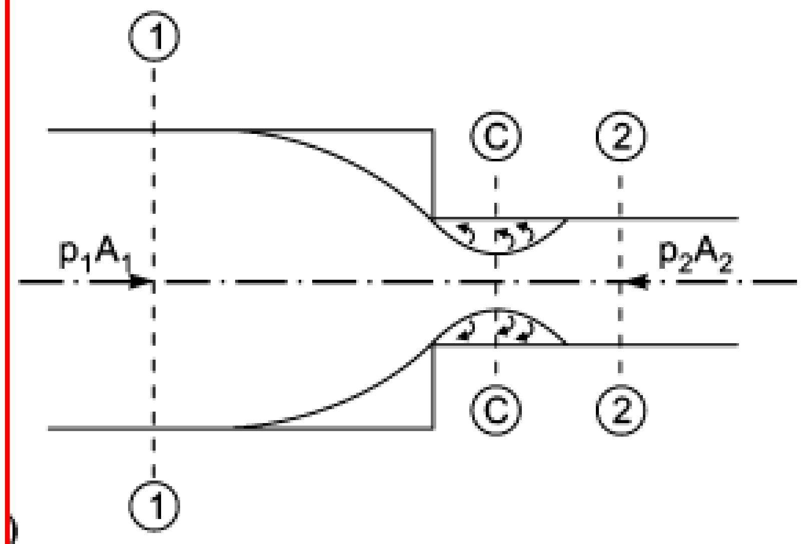


Fig. 11.2 *Sudden contraction.*

Bend loss

$$(h_l)_{Sudden\ entry} = \frac{k.V^2}{2g}$$

Angle of bend θ increases = **k increases**

Radius of pipe θ increases = **k decreases**

Reduce of curvature of pipe Increase **= K decreases**

Problem 11.8 *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Dia. of smaller pipe, $D_1 = 200$ mm = 0.20 m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400$ mm = 0.4 m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250$ litres/s = 0.25 m^3/s

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96$ m/s

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99$ m/s

Loss of head due to enlargement is given by

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \mathbf{1.816 \text{ m of water. Ans.}}$$

Problem 11.9 *At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.* (J.N.T.U., S 2002)

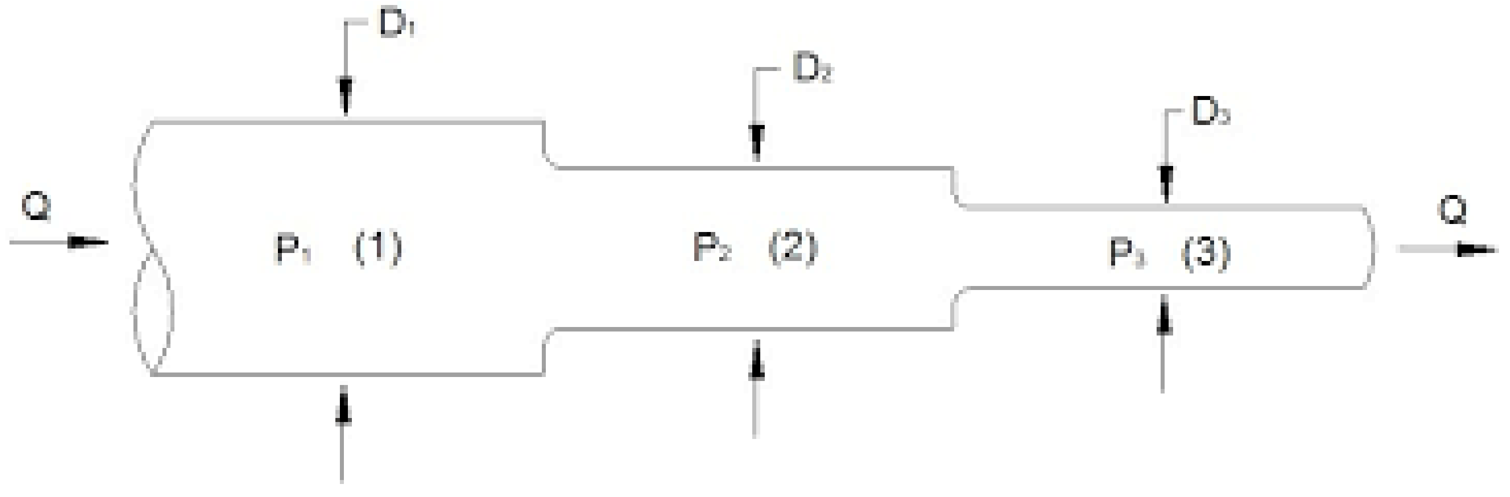
Problem 11.10 *The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine :*

(*i*) *loss of head due to sudden enlargement,* (*ii*) *pressure intensity in the large pipe,*

(*iii*) *power lost due to enlargement.*

Problem 11.11 *A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine the rate of flow of water.*

Pipes in series and pipes in parallel:



Pipes in series :

a) $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q_6 = Q_n$

b) $h_l = h_{l1} + h_{l2} + h_{l2} + \ldots\ldots\ldots + h_{ln}$

$$\frac{fL_1Q^2}{12D_e^5} = \frac{fL_1Q^2}{12D_1^5} + \frac{fL_2Q^2}{12D_2^5} + \frac{fL_3Q^2}{12D_3^5} + \frac{fL_4Q^2}{12D_4^5} + \frac{fL_5Q^2}{12D_5^5}$$

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \frac{L_4}{D_4^5} + \frac{L_5}{D_5^5}$$

Problem 11.31 *Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find the diameter of the single pipe.*

Length of pipe 1, $L_1 = 800$ m and dia., $d_1 = 500$ mm $= 0.5$ m
Length of pipe 2, $L_2 = 500$ m and dia., $d_2 = 400$ mm $= 0.4$ m
Length of pipe 3, $L_3 = 400$ m and dia., $d_3 = 300$ mm $= 0.3$ m

Length of single pipe, $L = 1700$ m

$$\frac{1700}{d^5} = \frac{800}{.5^5} + \frac{500}{.4^5} + \frac{400}{0.3^5}$$

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \frac{L_4}{D_4^5} + \frac{L_5}{D_5^5}$$

$$= 25600 + 48828.125 + 164609 = 239037$$

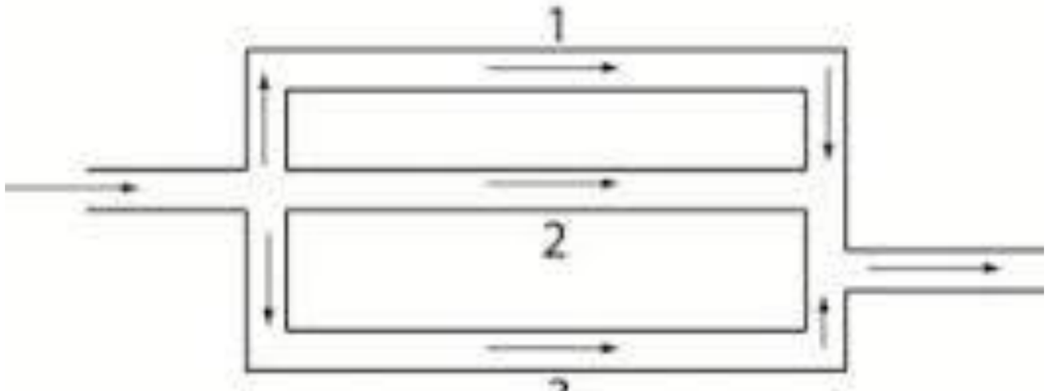
$$d^5 = \frac{1700}{239037} = .007118$$

$d = (.007188)^{0.2} = 0.3718 =$ **371.8 mm. Ans.**

Pipes in parallel:

a) $Q_n = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6$

b) $h_l = h_{l1} = h_{l2} = h_{l2} = \dots\dots\dots = h_{ln}$



Equivalent pipes : having same as that of compound pipes A equivalent pipe to a compound pipe is a pipe of uniform diameter having same discharge and same head loss at that of the compound pipe.

$$h_l = \frac{fL_1(Q/n)^2}{12D^5} = \frac{fL_1Q^2}{12D_e^5}$$

$$\frac{L_e}{n^2D^5} = \frac{L_e}{D_e^5}$$

Problem 11.32 *A main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. 11.17. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m*^{3}*/s. The co-efficient of friction for each parallel pipe is same and equal to .005.*

Length of pipe 1, $L_1 = 2000$ m
Dia. of pipe 1, $d_1 = 1.0$ m
Length of pipe 2, $L_2 = 2000$ m
Dia. of pipe 2, $d_2 = 0.8$ m
Total flow, $Q = 3.0$ m^3/s

$f_1 = f_2 = f = .005$
Q_1 = discharge in pipe 1
Q_2 = discharge in pipe 2

$Q = Q_1 + Q_2 = 3.0$

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\frac{4 \times .005 \times 2000 \times V_1}{1.0 \times 2 \times 9.81} = \frac{4 \times .005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$$

$$Q_1 = \frac{\pi}{4}\, d_1{}^2 \times V_1 = \frac{\pi}{4}\,(1)^2 \times \frac{V_2}{.894}$$

$$Q_2 = \frac{\pi}{4}\, d_2{}^2 \times V_2 = \frac{\pi}{4}\,(.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$$

$$Q = Q_1 + Q_2 = 3.0$$

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times .64\, V_2 = 3.0$$

$$V = \frac{3.0}{1.3811} \quad = 2.17 \text{ m/s.}$$

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

$$Q_1 = \frac{\pi}{4} d_1^{\,2} \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = \mathbf{1.906 \text{ m}^3\text{/s. Ans.}}$$

$$Q_2 = Q - Q_1 = 3.0 - 1.906 = \mathbf{1.094 \text{ m}^3\text{/s. Ans.}}$$

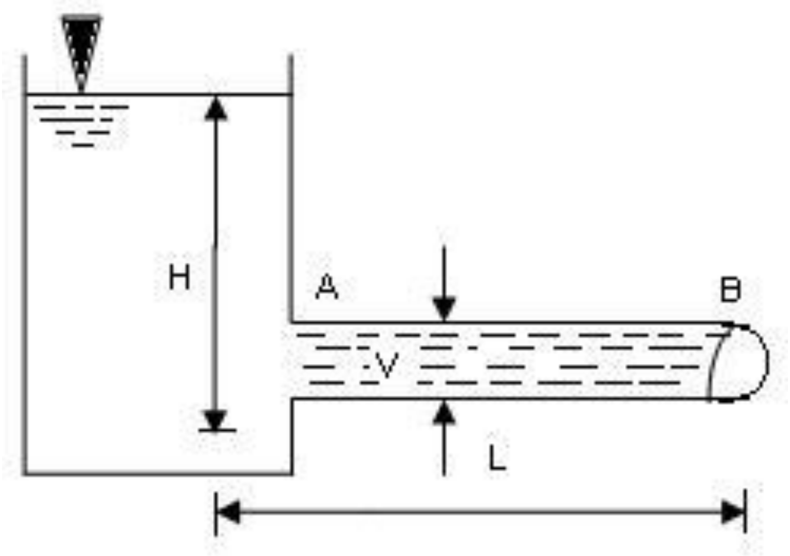
Power transmission through the pipe :

$$P_{theoritical} = P_{th} = \omega QH$$

$$P_{actual} = P_{act} = \omega Q(H - H_L)$$

$$\eta_{Power\ transmission} = \frac{P_{actual}}{P_{theoritical}} = \frac{P_{act}}{P_{th}}$$

$$\eta_{Power\ transmission} = \frac{H - H_L}{H}$$



Condition for maximum Power transmission :

$$P_{actual} = P_{act} = \omega Q(H - H_L)$$

$$\frac{dP_{actual}}{dQ} = 0$$

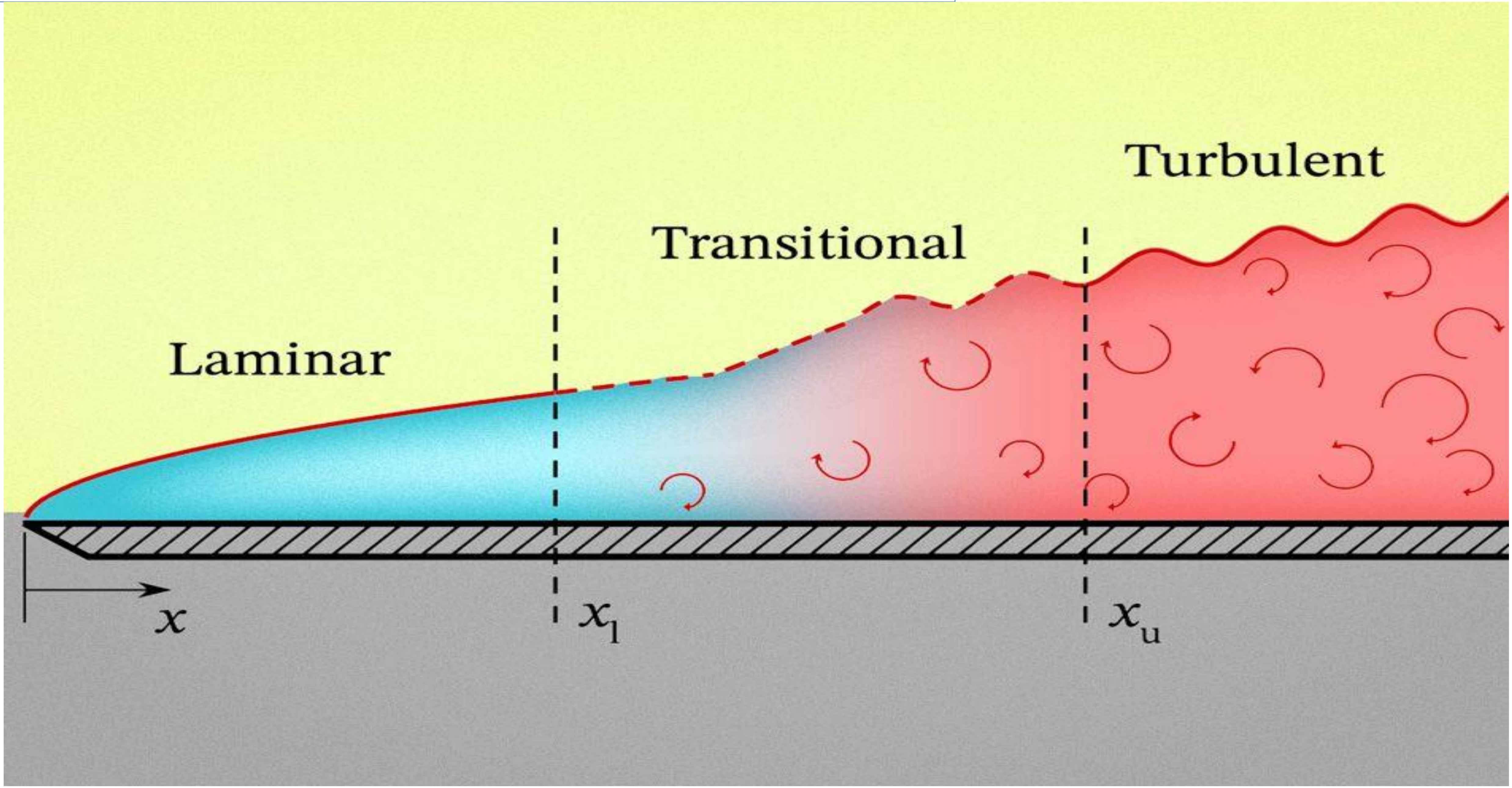
$$H = 3H_L$$

Maximum η of power transmission

$$\eta_{Power\ transmission} = \frac{H - H_L}{H}$$

$$= \frac{3H_L - H_L}{3H_L} = 66.67\ \%$$

$$\eta_{Power\ transmission} = 66.67\ \%$$


Turbulent
Transitional
Laminar
x
x_l
x_u