

ENGINEERING CURVES

Part- I {Conic Sections}

ELLIPSE

1. Concentric Circle Method
2. Rectangle Method
3. Oblong Method
4. Arcs of Circle Method
5. Rhombus Method
6. Basic Locus Method
(Directrix – focus)

PARABOLA

1. Rectangle Method
2. Method of Tangents
(Triangle Method)
3. Basic Locus Method
(Directrix – focus)

HYPERBOLA

1. Rectangular Hyperbola
(coordinates given)
2. Rectangular Hyperbola
(P-V diagram - Equation given)
3. Basic Locus Method
(Directrix – focus)

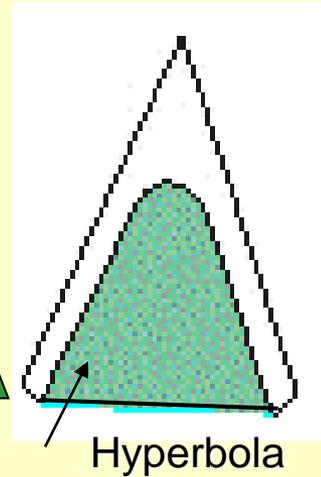
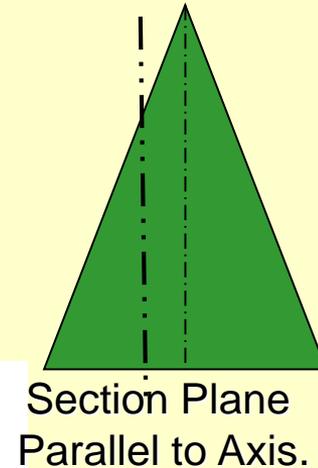
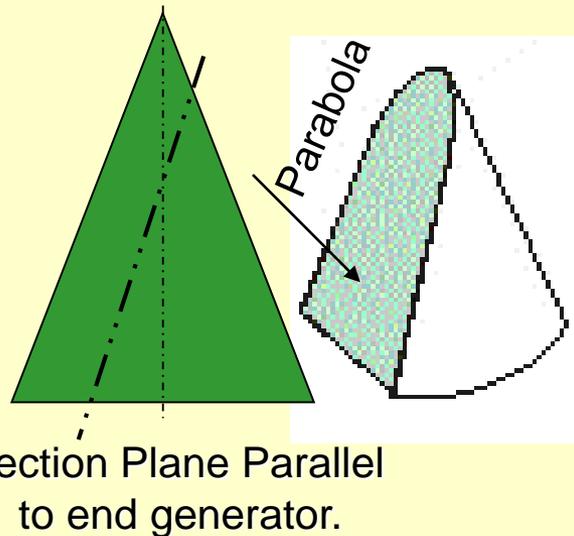
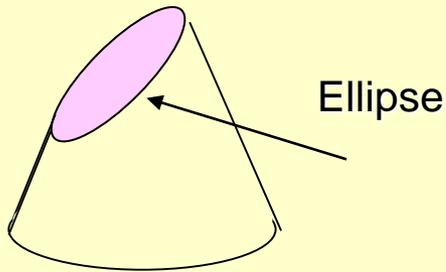
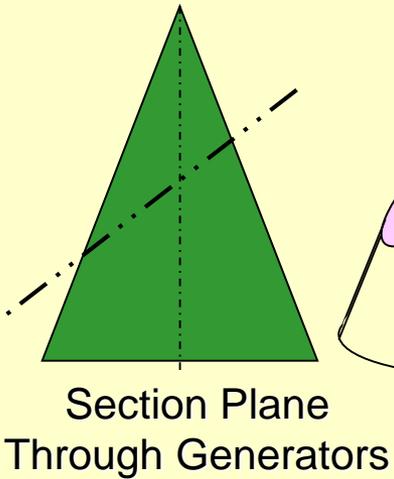
Methods of Drawing
Tangents & Normals
To These Curves.

CONIC SECTIONS

**ELLIPSE, PARABOLA AND HYPERBOLA ARE CALLED CONIC SECTIONS
BECAUSE**

**THESE CURVES APPEAR ON THE SURFACE OF A CONE
WHEN IT IS CUT BY SOME TYPICAL CUTTING PLANES.**

**OBSERVE
ILLUSTRATIONS
GIVEN BELOW..**



COMMON DEFINATION OF ELLIPSE, PARABOLA & HYPERBOLA:

These are the loci of points moving in a plane such that the ratio of it's distances from a *fixed point* And a *fixed line* always remains constant.

The Ratio is called **ECCENTRICITY. (E)**

- A) For Ellipse $E < 1$
- B) For Parabola $E = 1$
- C) For Hyperbola $E > 1$

Refer Problem nos. 6. 9 & 12

SECOND DEFINATION OF AN ELLIPSE:-

It is a locus of a point moving in a plane such that the **SUM** of it's distances from **TWO** fixed points always remains constant.

{ And this *sum equals* to the length of *major axis*. }

These **TWO** fixed points are **FOCUS 1 & FOCUS 2**

Refer Problem no.4
Ellipse by Arcs of Circles Method.

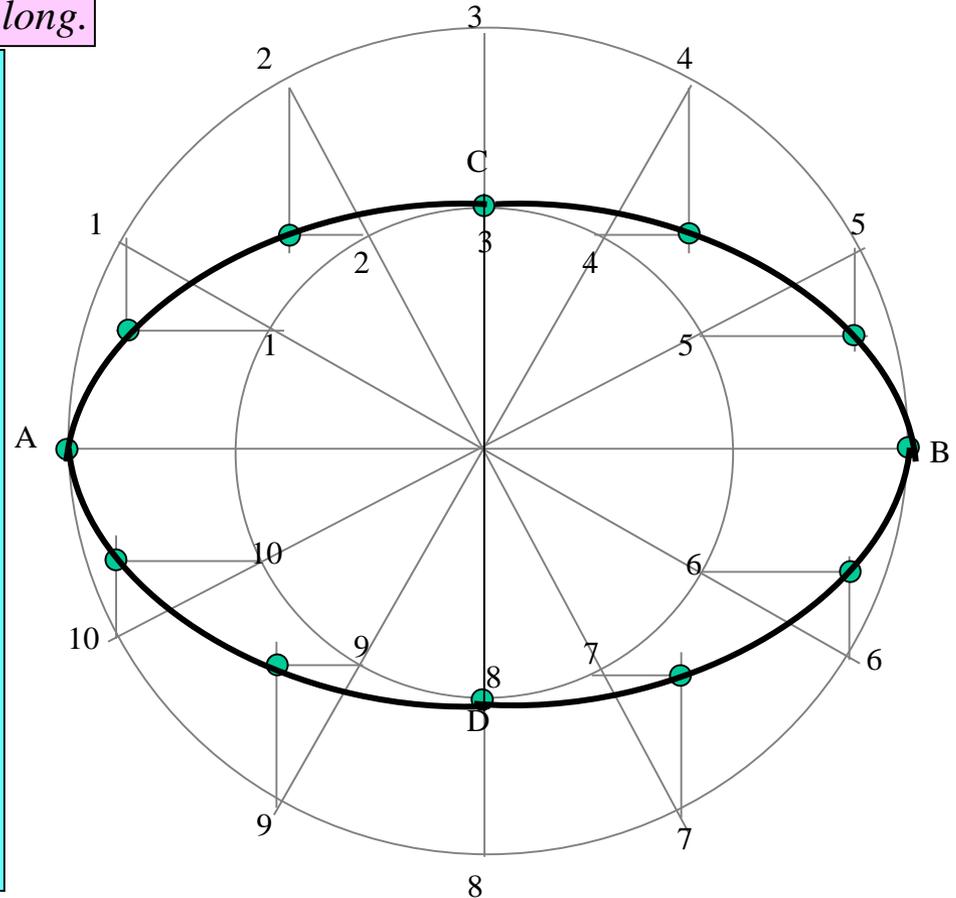
Problem 1 :-

Draw ellipse by **concentric circle method**.

Take major axis 100 mm and minor axis 80 mm long.

Steps:

1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
3. Divide both circles in 12 equal parts & name as shown.
4. From all points of outer circle draw vertical lines downwards and upwards respectively.
5. From all points of inner circle draw horizontal lines to intersect those vertical lines.
6. Mark all intersecting points properly as those are the points on ellipse.
7. Join all these points along with the ends of both axes in smooth possible curve. It is required ellipse.



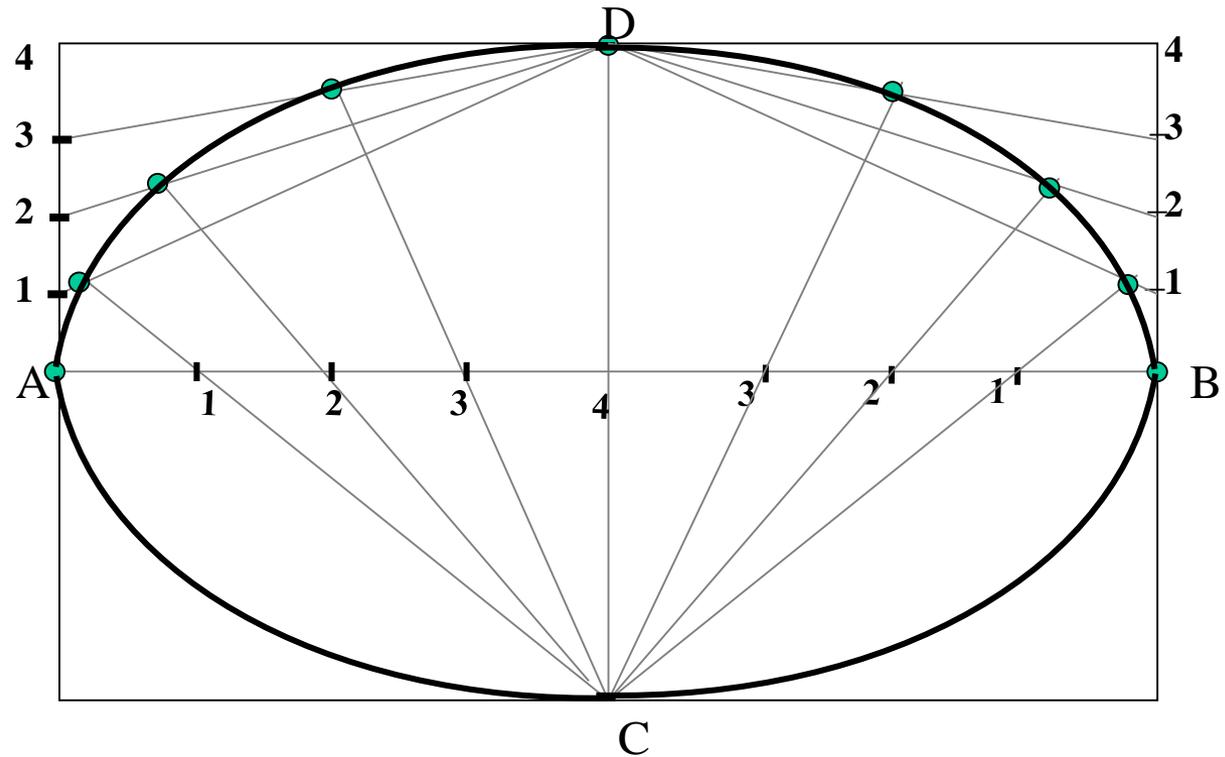
Steps:

- 1 Draw a rectangle taking major and minor axes as sides.
 2. In this rectangle draw both axes as perpendicular bisectors of each other..
 3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts.(here divided in four parts)
 4. Name those as shown..
 5. Now join all vertical points 1,2,3,4, to the upper end of minor axis. And all horizontal points i.e.1,2,3,4 to the lower end of minor axis.
 6. Then extend C-1 line upto D-1 and mark that point. Similarly extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
 7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side part.along with lower half of the rectangle.Join all points in smooth curve.
- It is required ellipse.

Problem 2

Draw ellipse by **Rectangle method**.

Take major axis 100 mm and minor axis 70 mm long.

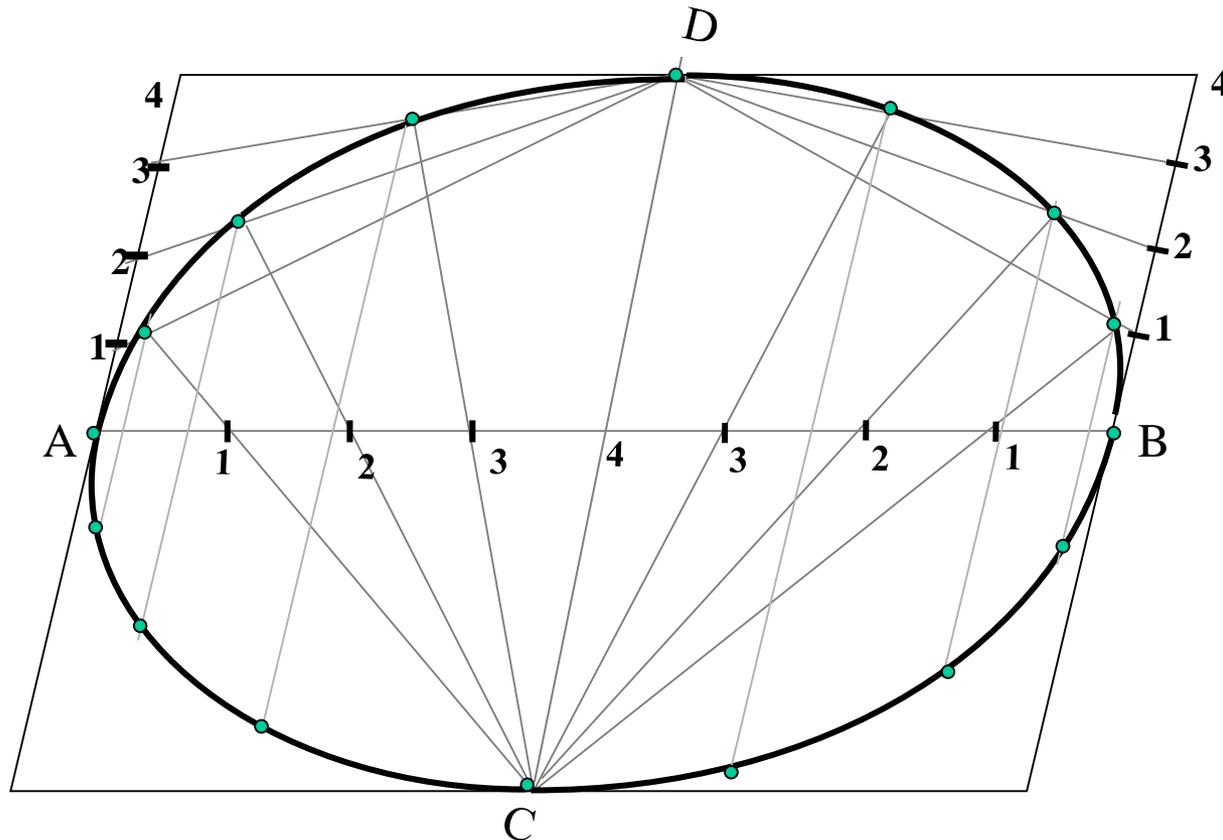


Problem 3:-

Draw ellipse by Oblong method.

Draw a parallelogram of 100 mm and 70 mm long sides with included angle of 75° . Inscribe Ellipse in it.

**STEPS ARE SIMILAR TO
THE PREVIOUS CASE
(RECTANGLE METHOD)
ONLY IN PLACE OF RECTANGLE,
HERE IS A PARALLELOGRAM.**



ELLIPSE

BY ARCS OF CIRCLE METHOD

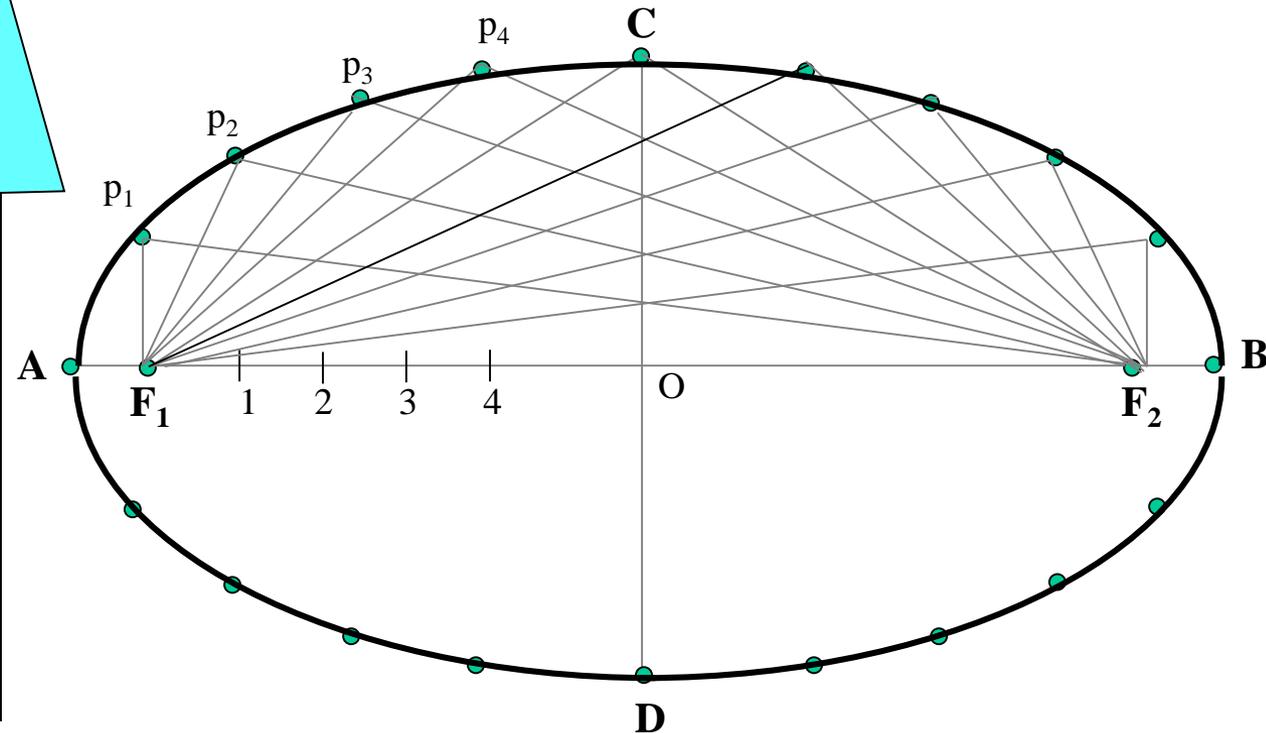
PROBLEM 4.

MAJOR AXIS AB & MINOR AXIS CD ARE 100 AND 70MM LONG RESPECTIVELY .DRAW ELLIPSE BY ARCS OF CIRCLES METHOD.

STEPS:

1. Draw both axes as usual. Name the ends & intersecting point
2. Taking AO distance I.e. half major axis, from C, mark F_1 & F_2 On AB . (focus 1 and 2.)
3. On line $F_1 - O$ taking any distance, mark points 1,2,3, & 4
4. Taking F_1 center, with distance A-1 draw an arc above AB and taking F_2 center, with B-1 distance cut this arc. Name the point p_1
5. Repeat this step with same centers but taking now A-2 & B-2 distances for drawing arcs. Name the point p_2
6. Similarly get all other P points. With same steps positions of P can be located below AB.
7. Join all points by smooth curve to get an ellipse/

As per the definition Ellipse is locus of point P moving in a plane such that the **SUM** of it's distances from two fixed points (F_1 & F_2) remains constant and equals to the length of major axis AB. (Note A . 1 + B . 1 = A . 2 + B . 2 = AB)



ELLIPSE

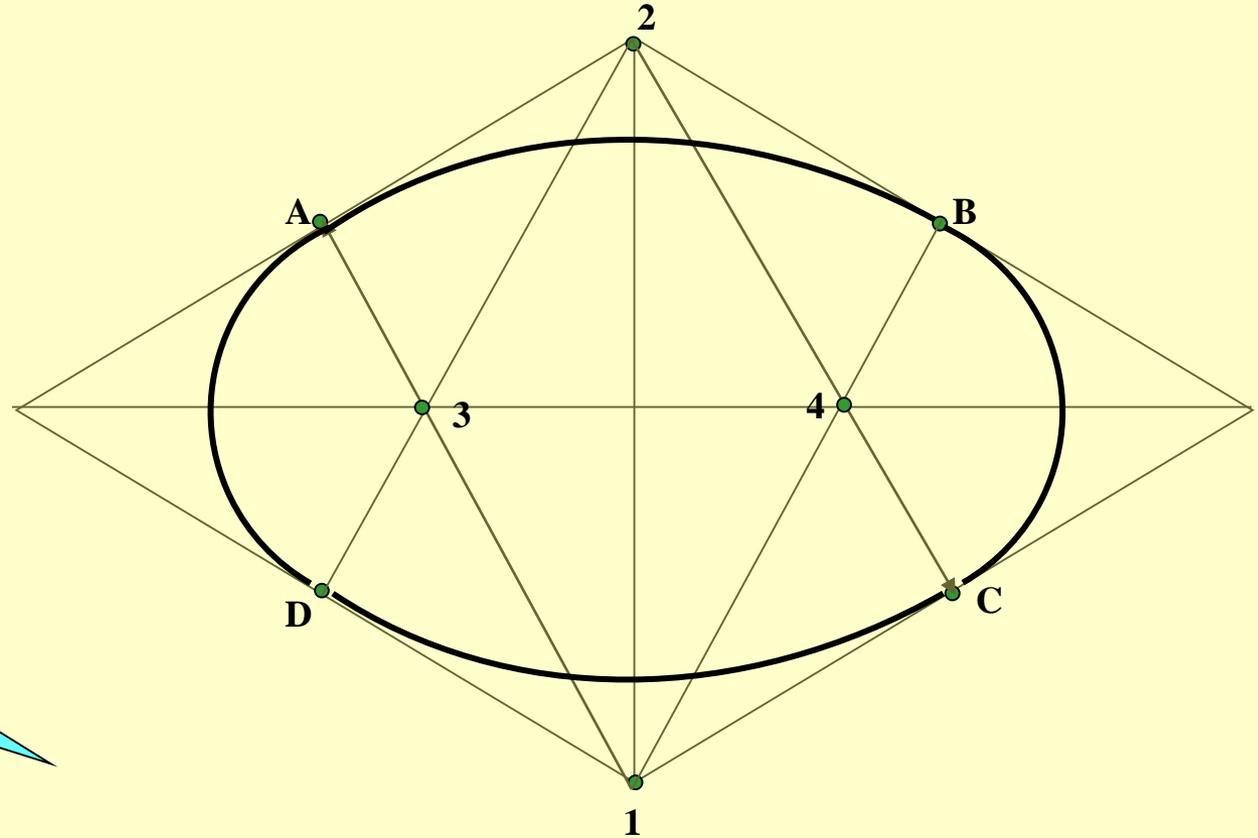
BY RHOMBUS METHOD

PROBLEM 5.

DRAW RHOMBUS OF 100 MM & 70 MM LONG DIAGONALS AND INSCRIBE AN ELLIPSE IN IT.

STEPS:

1. Draw rhombus of given dimensions.
2. Mark mid points of all sides & name Those A,B,C,& D
3. Join these points to the ends of smaller diagonals.
4. Mark points 1,2,3,4 as four centers.
5. Taking 1 as center and 1-A radius draw an arc AB.
6. Take 2 as center draw an arc CD.
7. Similarly taking 3 & 4 as centers and 3-D radius draw arcs DA & BC.



ELLIPSE

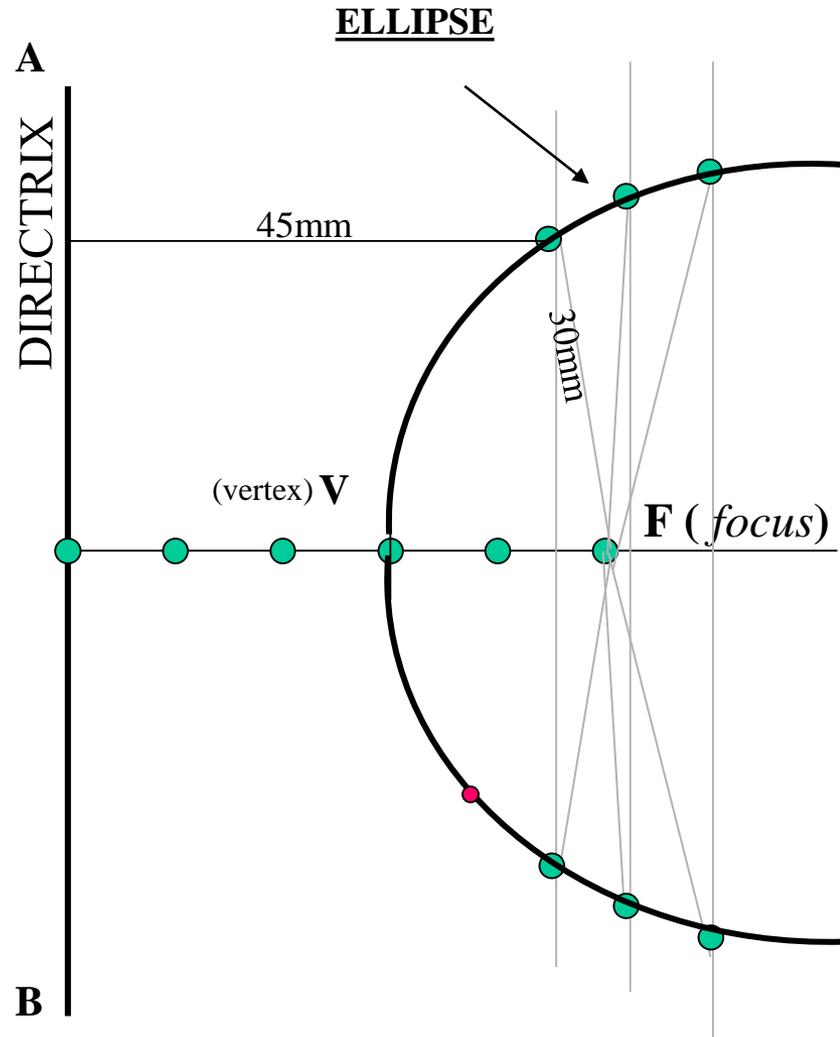
DIRECTRIX-FOCUS METHOD

PROBLEM 6:- POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE **RATIO** OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO **2/3** DRAW LOCUS OF POINT P. { **ECCENTRICITY = 2/3** }

STEPS:

1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2nd part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB $2/3$ i.e $20/30$
4. Form more points giving same ratio such as $30/45$, $40/60$, $50/75$ etc.
5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.

This is required locus of P. It is an ELLIPSE.

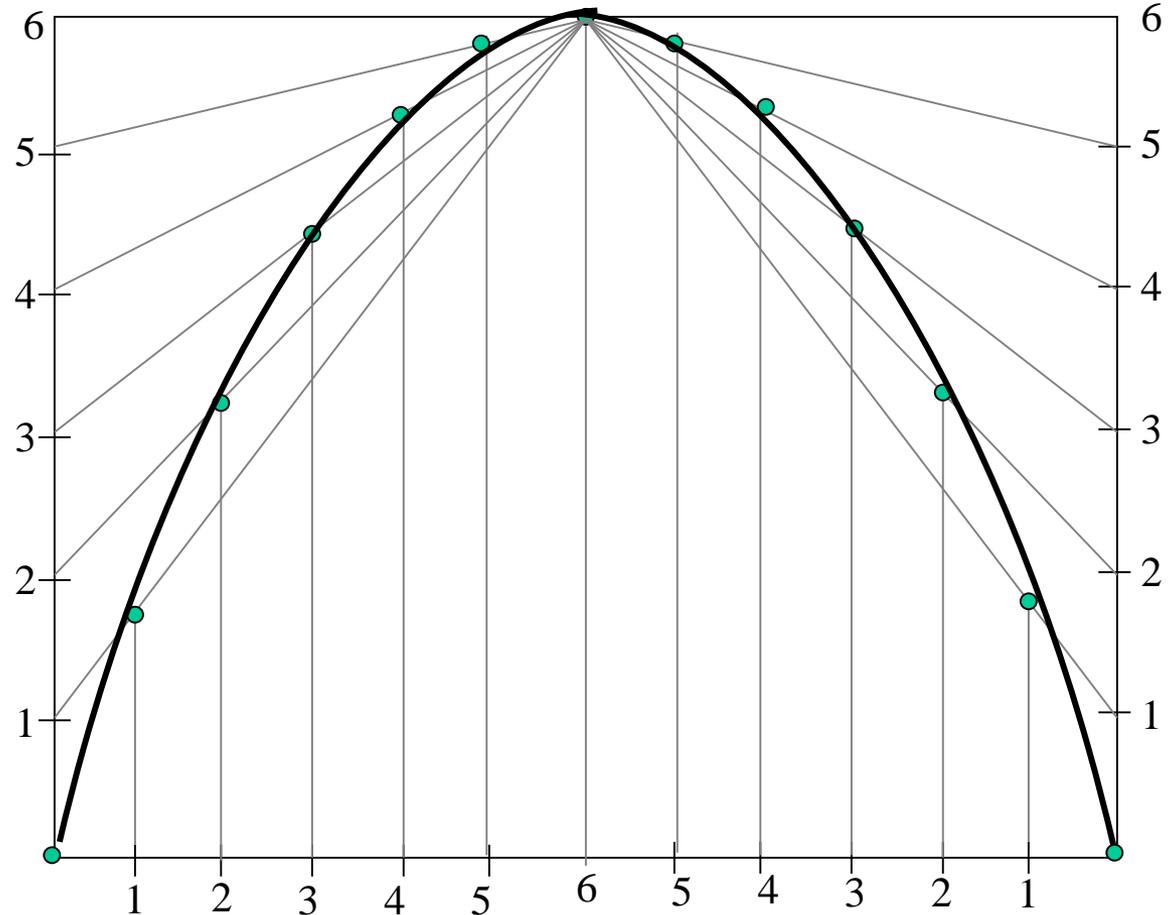


PROBLEM 7: A BALL THROWN IN AIR ATTAINS 100 M HEIGHT AND COVERS HORIZONTAL DISTANCE 150 M ON GROUND.
Draw the path of the ball (projectile)-

PARABOLA RECTANGLE METHOD

STEPS:

1. Draw rectangle of above size and divide it in two equal vertical parts
 2. Consider left part for construction. Divide height and length in equal number of parts and name those 1,2,3,4,5 & 6
 3. Join vertical 1,2,3,4,5 & 6 to the top center of rectangle
 4. Similarly draw upward vertical lines from horizontal 1,2,3,4,5. And wherever these lines intersect previously drawn inclined lines in sequence Mark those points and further join in smooth possible curve.
 5. Repeat the construction on right side rectangle also. Join all in sequence.
- This locus is Parabola.**



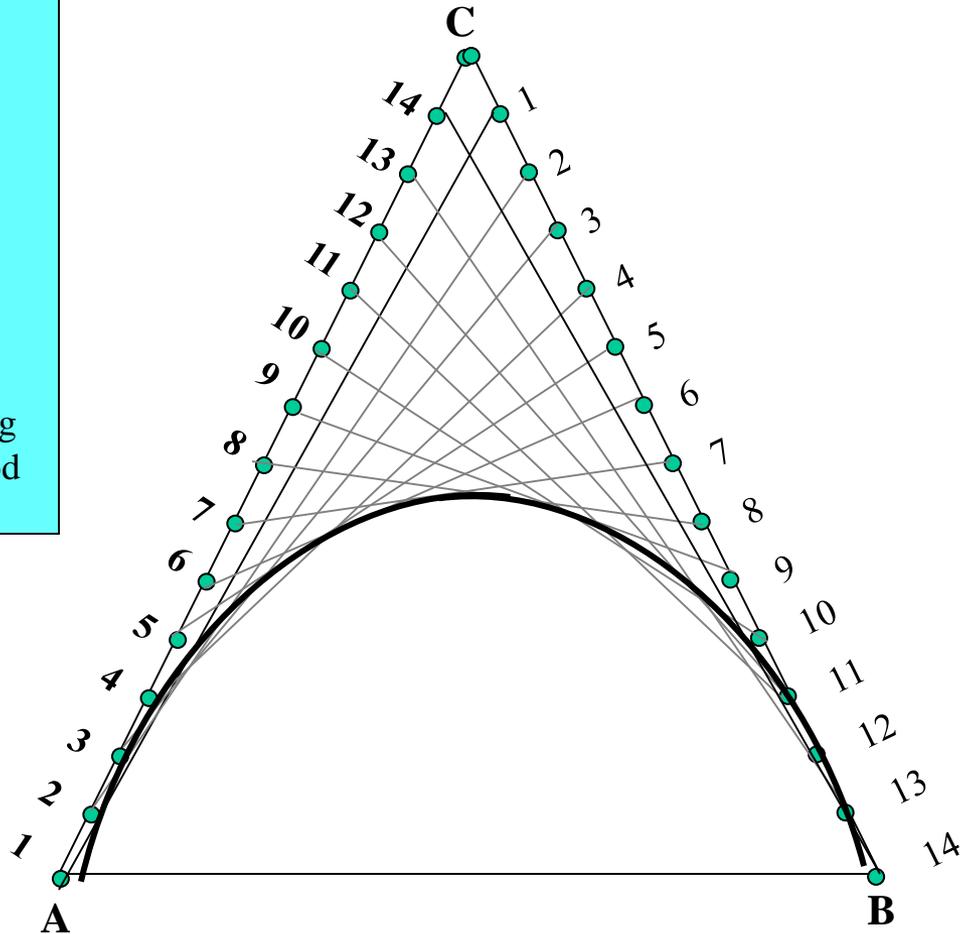
PARABOLA

METHOD OF TANGENTS

Problem no.8: Draw an isosceles triangle of 100 mm long base and 110 mm long altitude. Inscribe a parabola in it by method of tangents.

Solution Steps:

1. Construct triangle as per the given dimensions.
2. Divide its both sides into same no. of equal parts.
3. Name the parts in ascending and descending manner, as shown.
4. Join 1-1, 2-2, 3-3 and so on.
5. Draw the curve as shown i.e. tangent to all these lines. The above all lines being tangents to the curve, it is called method of tangents.



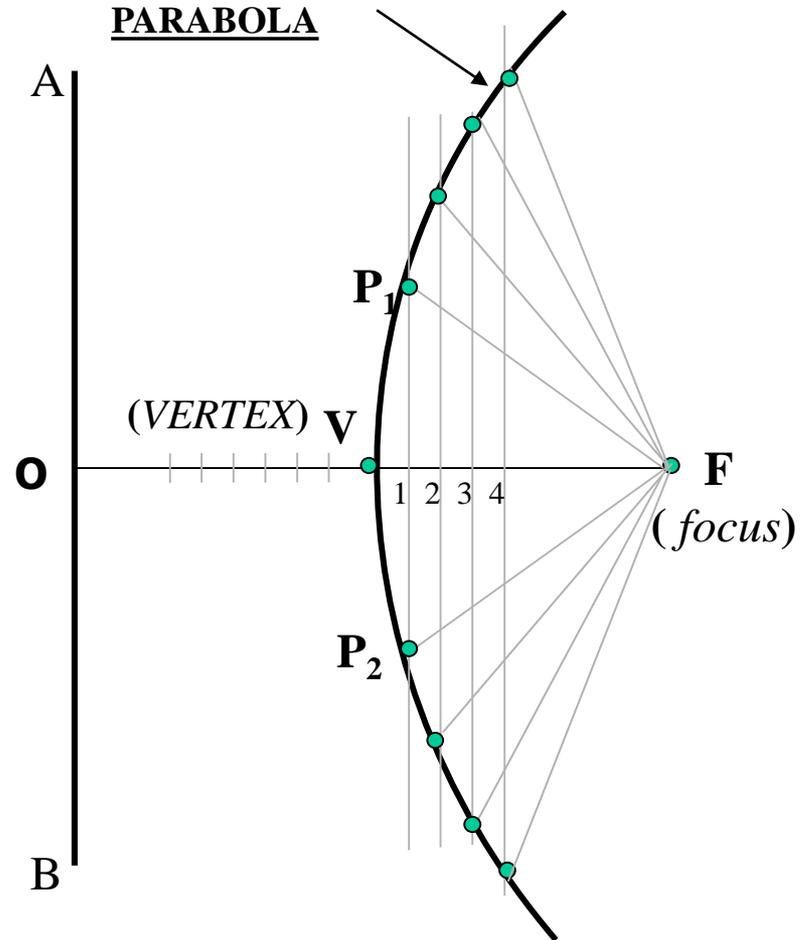
PROBLEM 9: Point F is 50 mm from a vertical straight line AB. Draw locus of point P, moving in a plane such that it always remains equidistant from point F and line AB.

PARABOLA DIRECTRIX-FOCUS METHOD

SOLUTION STEPS:

1. Locate center of line, perpendicular to AB from point F. This will be initial point P and also the vertex.
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.
3. Mark 5 mm distance to its left of P and name it 1.
4. Take O-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point P_1 and lower point P_2 . ($FP_1=O1$)
5. Similarly repeat this process by taking again 5mm to right and left and locate P_3P_4 .
6. Join all these points in smooth curve.

It will be the locus of P equidistance from line AB and fixed point F.

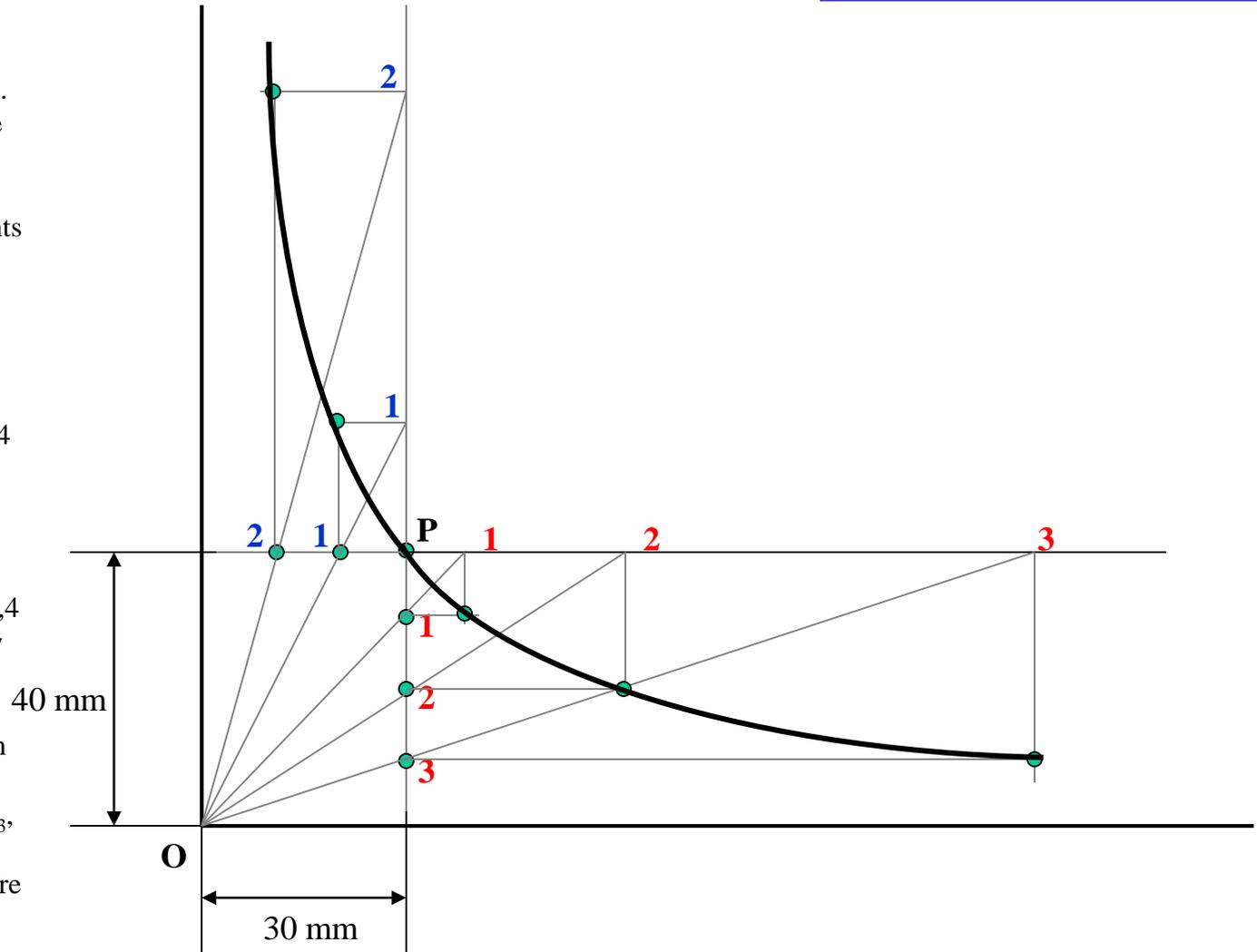


Problem No.10: Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.

HYPERBOLA THROUGH A POINT OF KNOWN CO-ORDINATES

Solution Steps:

- 1) Extend horizontal line from P to right side.
- 2) Extend vertical line from P upward.
- 3) On horizontal line from P, mark some points taking any distance and name them after P-1, 2,3,4 etc.
- 4) Join 1-2-3-4 points to pole O. Let them cut part [P-B] also at 1,2,3,4 points.
- 5) From horizontal 1,2,3,4 draw vertical lines downwards and
- 6) From vertical 1,2,3,4 points [from P-B] draw horizontal lines.
- 7) Line from 1 horizontal and line from 1 vertical will meet at P₁. Similarly mark P₂, P₃, P₄ points.
- 8) Repeat the procedure by marking four points on upward vertical line from P and joining all those to pole O. Name this points P₆, P₇, P₈ etc. and join them by smooth curve.



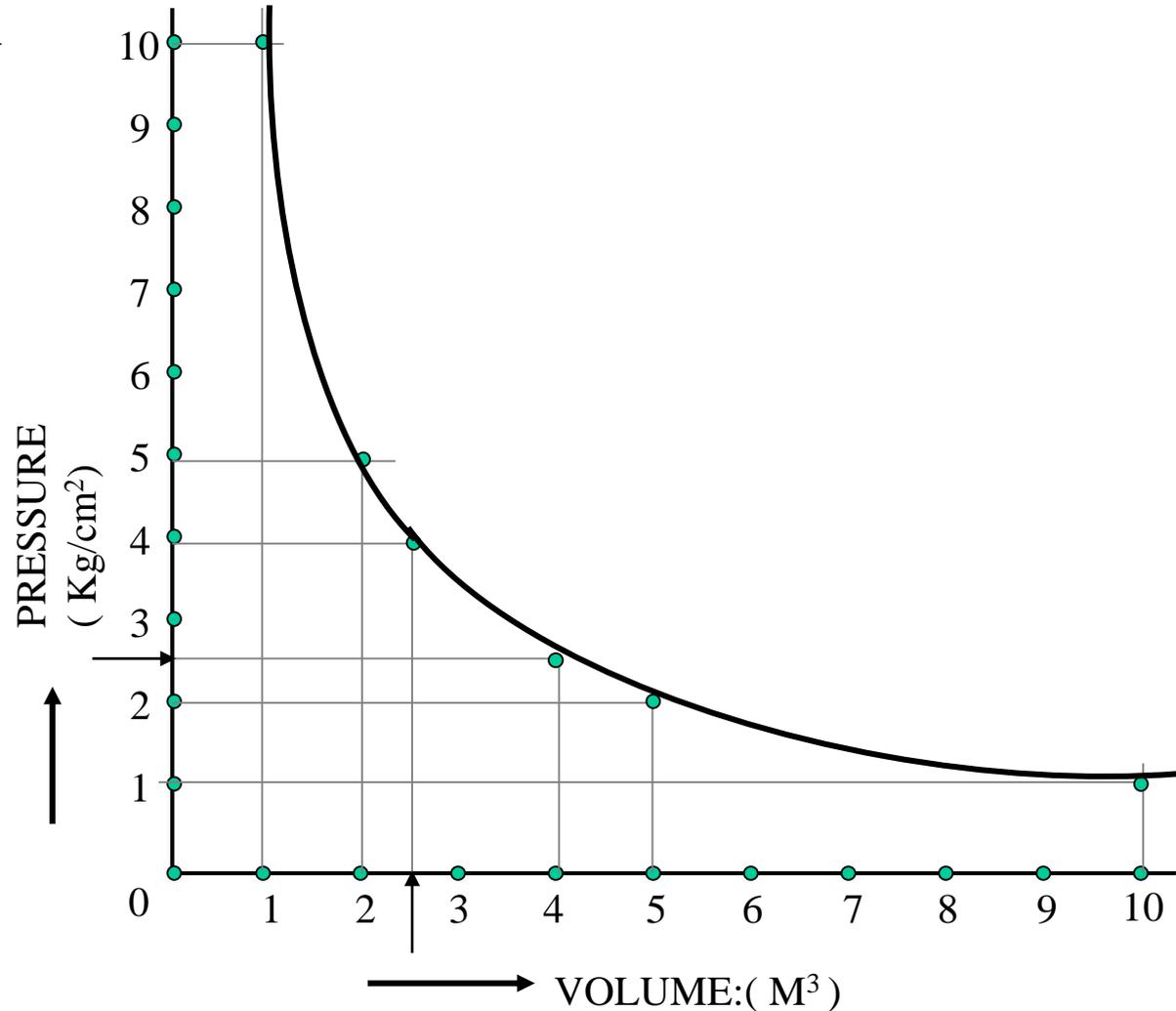
Problem no.11: A sample of gas is expanded in a cylinder from 10 unit pressure to 1 unit pressure. Expansion follows law $PV=Constant$. If initial volume being 1 unit, draw the curve of expansion. Also Name the curve.

HYPERBOLA P-V DIAGRAM

Form a table giving few more values of P & V

$P \times V = C$
$10 \times 1 = 10$
$5 \times 2 = 10$
$4 \times 2.5 = 10$
$2.5 \times 4 = 10$
$2 \times 5 = 10$
$1 \times 10 = 10$

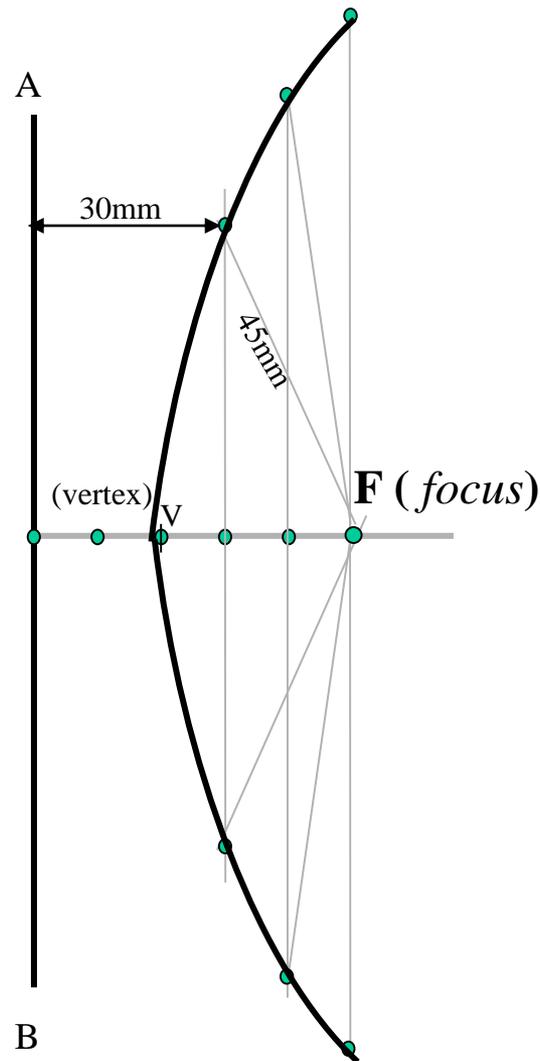
Now draw a Graph of Pressure against Volume.
It is a PV Diagram and it is Hyperbola.
Take pressure on vertical axis and Volume on horizontal axis.



PROBLEM 12:- POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE **RATIO** OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO **2/3** DRAW LOCUS OF POINT P. { **ECCENTRICITY = 2/3** }

STEPS:

1. Draw a vertical line AB and point F 50 mm from it.
 2. Divide 50 mm distance in 5 parts.
 3. Name 2nd part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB $2/3$ i.e $20/30$
 4. Form more points giving same ratio such as $30/45$, $40/60$, $50/75$ etc.
 5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
 6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
 7. Join these points through V in smooth curve.
- This is required locus of P. It is an ELLIPSE.



ENGINEERING CURVES

Part-II

(Point undergoing two types of displacements)

INVOLUTE

1. Involute of a circle

a)String Length = πD

b)String Length $> \pi D$

c)String Length $< \pi D$

2. Pole having Composite shape.

3. Rod Rolling over a Semicircular Pole.

CYCLOID

1. General Cycloid

2. Trochoid (superior)

3. Trochoid (Inferior)

4. Epi-Cycloid

5. Hypo-Cycloid

SPIRAL

1. Spiral of One Convolution.

2. Spiral of Two Convolution.

HELIX

1. On Cylinder

2. On a Cone

AND

Methods of Drawing Tangents & Normals To These Curves.

DEFINITIONS



CYCLOID:

IT IS A LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A STRAIGHT LINE PATH.

INVOLUTE:

IT IS A LOCUS OF A FREE END OF A STRING WHEN IT IS WOUND ROUND A CIRCULAR POLE

SPIRAL:

IT IS A CURVE GENERATED BY A POINT WHICH REVOLVES AROUND A FIXED POINT AND AT THE SAME MOVES TOWARDS IT.

HELIX:

IT IS A CURVE GENERATED BY A POINT WHICH MOVES AROUND THE SURFACE OF A RIGHT CIRCULAR CYLINDER / CONE AND AT THE SAME TIME ADVANCES IN AXIAL DIRECTION AT A SPEED BEARING A CONSTANT RATIO TO THE SPEED OF ROTATION.

(for problems refer topic Development of surfaces)

SUPERIORTROCHOID:

IF THE POINT IN THE DEFINATION OF CYCLOID IS OUTSIDE THE CIRCLE

INFERIOR TROCHOID.:

IF IT IS INSIDE THE CIRCLE

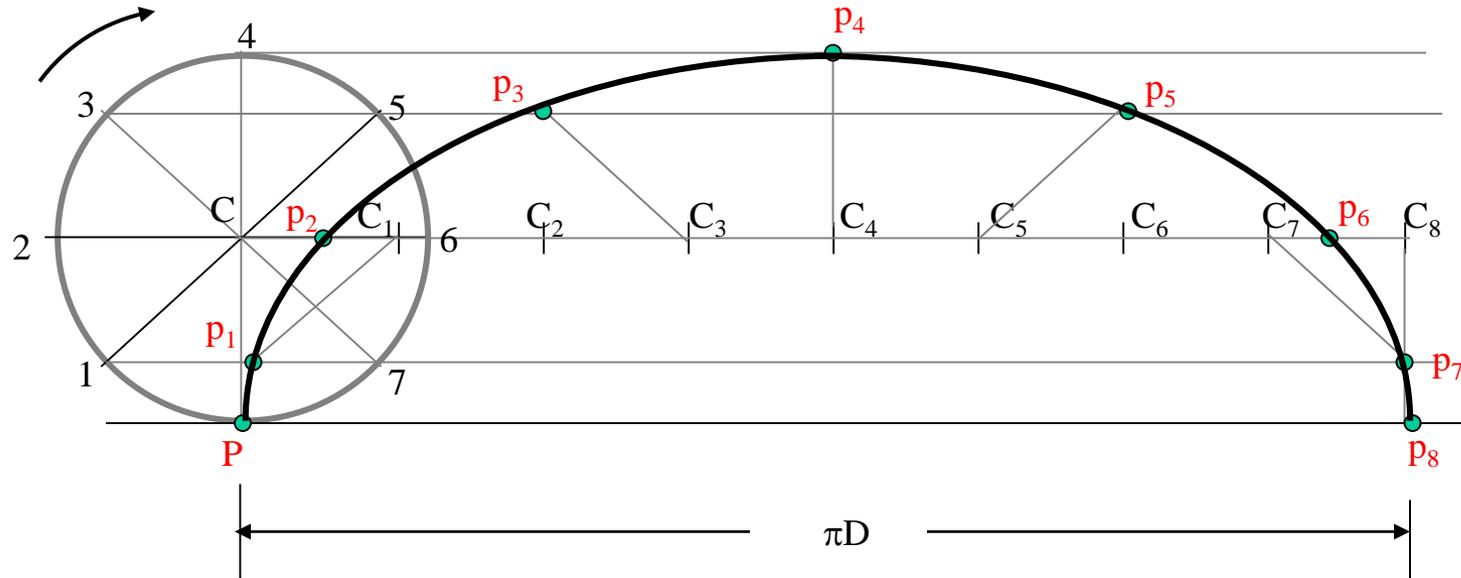
EPI-CYCLOID

IF THE CIRCLE IS ROLLING ON ANOTHER CIRCLE FROM OUTSIDE

HYP0-CYCLOID,

IF THE CIRCLE IS ROLLING FROM INSIDE THE OTHER CIRCLE,

PROBLEM 22: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

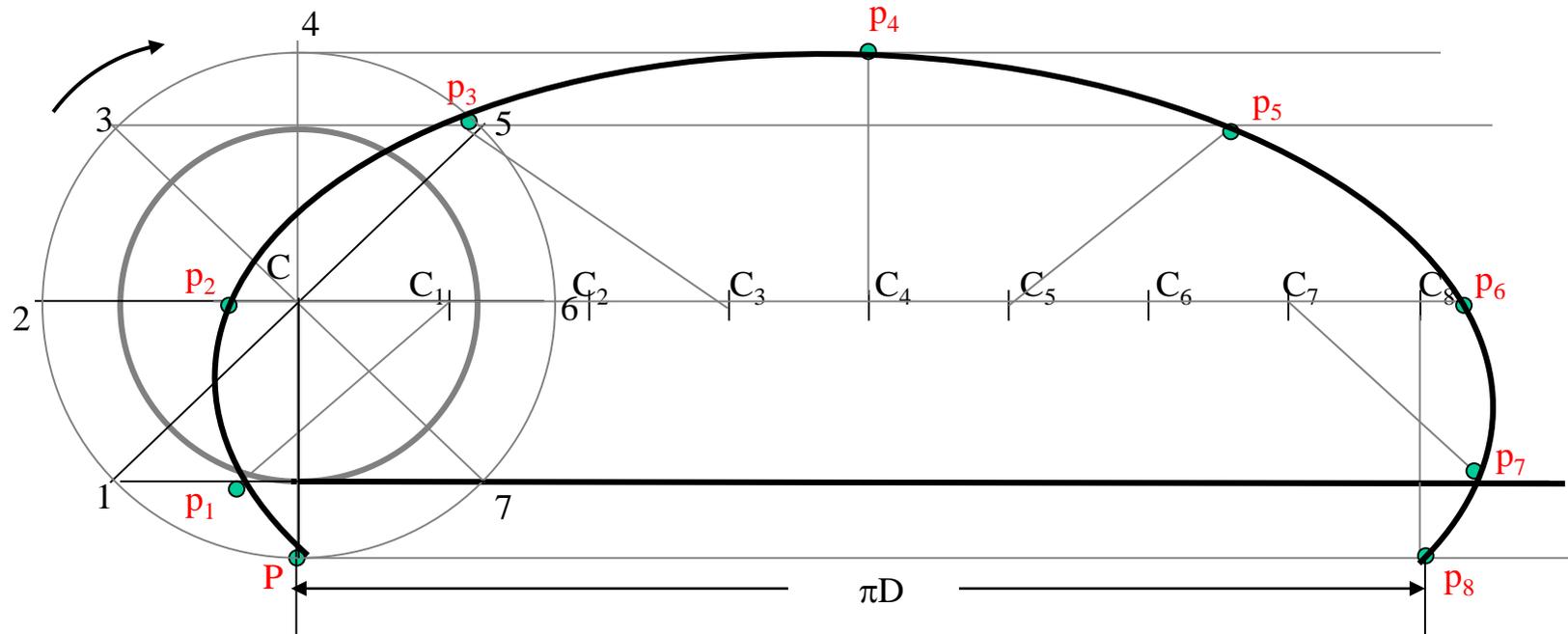


Solution Steps:

- 1) From center C draw a horizontal line equal to πD distance.
- 2) Divide πD distance into 8 number of equal parts and name them C1, C2, C3__ etc.
- 3) Divide the circle also into 8 number of equal parts and in clock wise direction, after P name 1, 2, 3 up to 8.
- 4) From all these points on circle draw horizontal lines. (parallel to locus of C)
- 5) With a fixed distance C-P in compass, C1 as center, mark a point on horizontal line from 1. Name it P.
- 6) Repeat this procedure from C2, C3, C4 upto C8 as centers. Mark points P2, P3, P4, P5 up to P8 on the horizontal lines drawn from 2, 3, 4, 5, 6, 7 respectively.
- 7) Join all these points by curve. **It is Cycloid.**

PROBLEM 23: DRAW LOCUS OF A POINT , 5 MM AWAY FROM THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

SUPERIOR TROCHOID

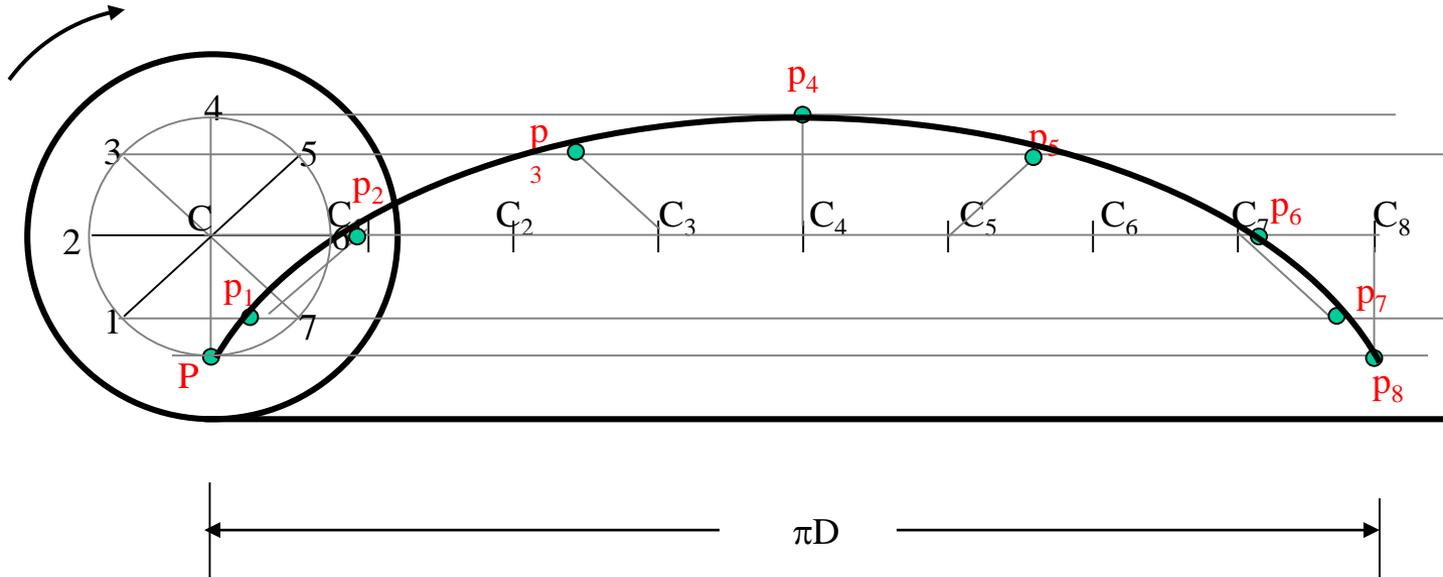


Solution Steps:

- 1) Draw circle of given diameter and draw a horizontal line from its center C of length πD and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
- 2) Draw circle by CP radius, as in this case CP is larger than radius of circle.
- 3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join
- 4) This curve is called **Superior Trochoid**.

PROBLEM 24: DRAW LOCUS OF A POINT , 5 MM INSIDE THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

INFERIOR TROCHOID



Solution Steps:

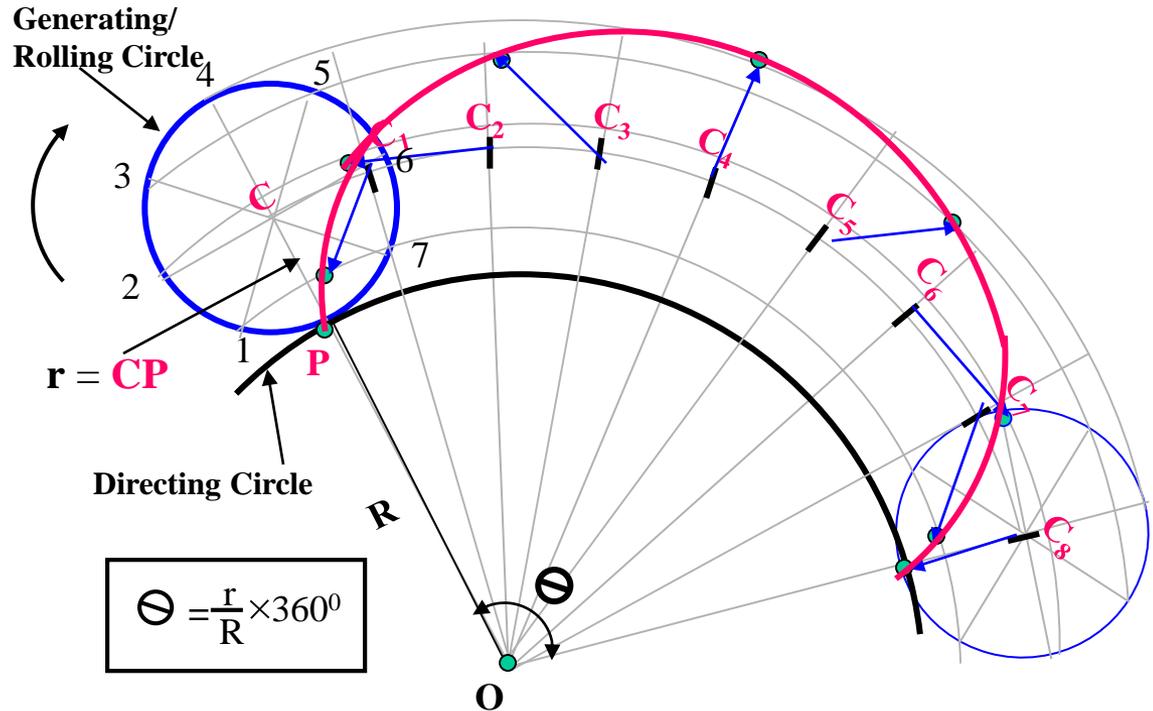
- 1) Draw circle of given diameter and draw a horizontal line from its center C of length πD and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
- 2) Draw circle by CP radius, as in this case CP is SHORTER than radius of circle.
- 3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join those in curvature.
- 4) This curve is called **Inferior Trochoid**.

PROBLEM 25: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A CURVED PATH. Take diameter of rolling Circle 50 mm And radius of directing circle i.e. curved path, 75 mm.

EPI CYCLOID :-

Solution Steps:

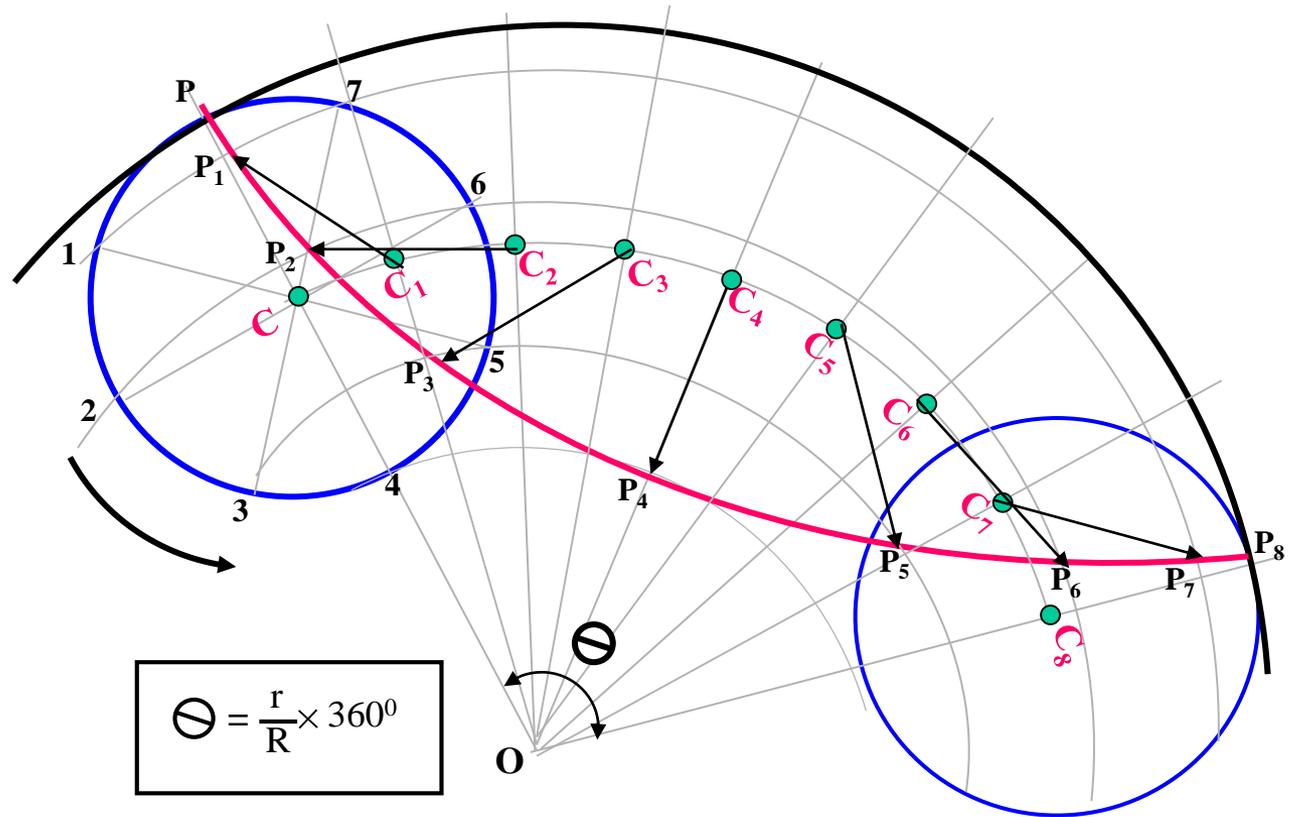
- 1) When smaller circle will roll on larger circle for one revolution it will cover πD distance on arc and it will be decided by included arc angle θ .
- 2) Calculate θ by formula $\theta = (r/R) \times 3600$.
- 3) Construct angle θ with radius OC and draw an arc by taking O as center OC as radius and form sector of angle θ .
- 4) Divide this sector into 8 number of equal angular parts. And from C onward name them C1, C2, C3 up to C8.
- 5) Divide smaller circle (Generating circle) also in 8 number of equal parts. And next to P in clockwise direction name those 1, 2, 3, up to 8.
- 6) With O as center, O-1 as radius draw an arc in the sector. Take O-2, O-3, O-4, O-5 up to O-8 distances with center O, draw all concentric arcs in sector. Take fixed distance C-P in compass, C1 center, cut arc of 1 at P1. Repeat procedure and locate P2, P3, P4, P5 upto P8 (as in cycloid) and join them by smooth curve. This is EPI - CYCLOID.



PROBLEM 26: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS FROM THE INSIDE OF A CURVED PATH. Take diameter of rolling circle 50 mm and radius of directing circle (curved path) 75 mm.

Solution Steps:

- 1) Smaller circle is rolling here, inside the larger circle. It has to rotate anticlockwise to move ahead.
- 2) Same steps should be taken as in case of EPI – CYCLOID. Only change is in numbering direction of 8 number of equal parts on the smaller circle.
- 3) From next to P in anticlockwise direction, name 1,2,3,4,5,6,7,8.
- 4) Further all steps are that of epi – cycloid. **This is called HYPO – CYCLOID.**



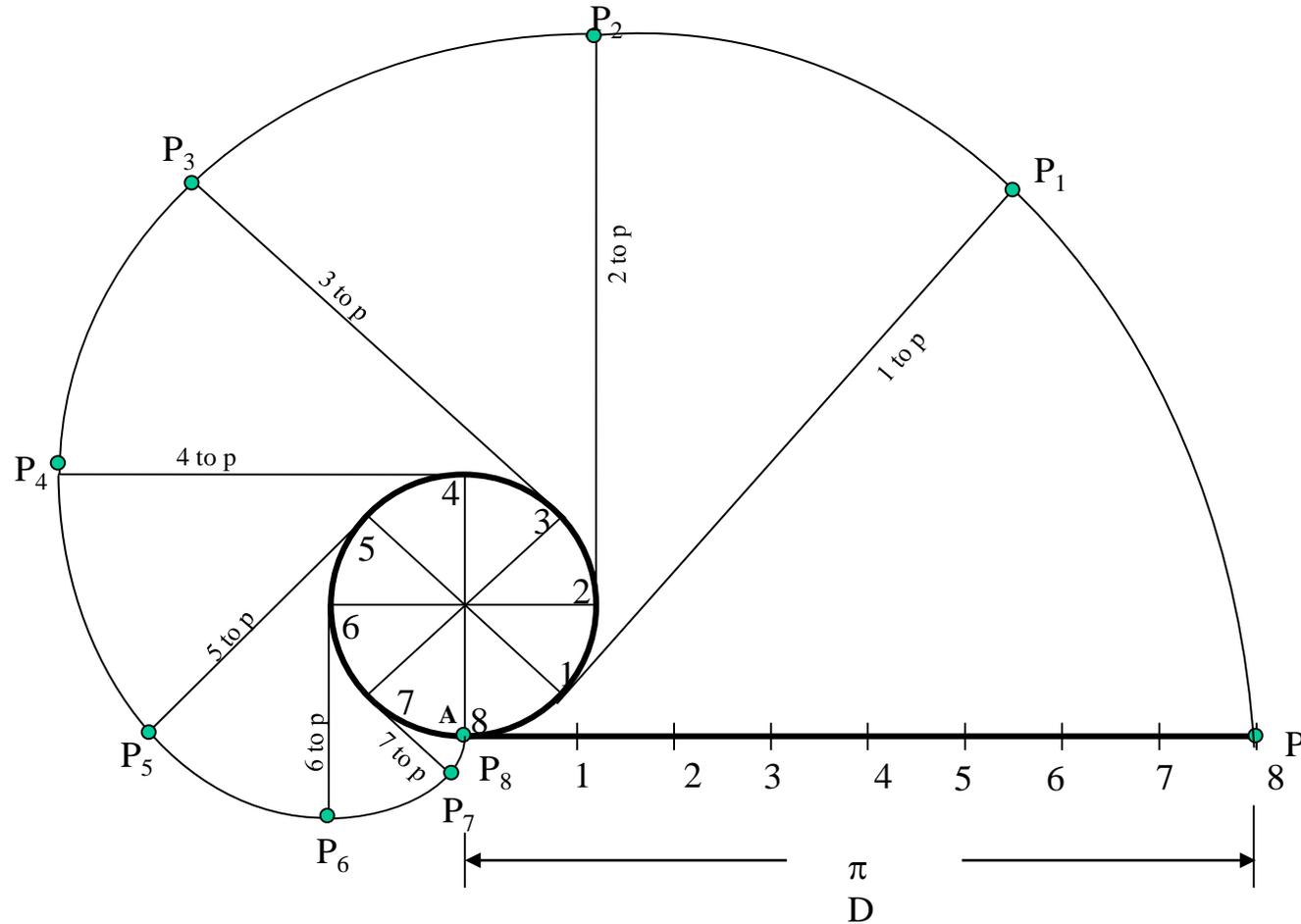
$$\Theta = \frac{r}{R} \times 360^\circ$$

OC = R (Radius of Directing Circle)
 CP = r (Radius of Generating Circle)

Problem no 17: Draw Involute of a circle.
 String length is equal to the circumference of circle.

Solution Steps:

- 1) Point or end P of string AP is exactly πD distance away from A. Means if this string is wound round the circle, it will completely cover given circle. B will meet A after winding.
- 2) Divide πD (AP) distance into 8 number of equal parts.
- 3) Divide circle also into 8 number of equal parts.
- 4) Name after A, 1, 2, 3, 4, etc. up to 8 on πD line AP as well as on circle (in anticlockwise direction).
- 5) To radius C-1, C-2, C-3 up to C-8 draw tangents (from 1,2,3,4,etc to circle).
- 6) Take distance 1 to P in compass and mark it on tangent from point 1 on circle (means one division less than distance AP).
- 7) Name this point P1
- 8) Take 2-B distance in compass and mark it on the tangent from point 2. Name it point P2.
- 9) Similarly take 3 to P, 4 to P, 5 to P up to 7 to P distance in compass and mark on respective tangents and locate P3, P4, P5 up to P8 (i.e. A) points and join them in smooth curve it is an INVOLUTE of a given circle.



INVOLUTE OF A CIRCLE

String length MORE than πD

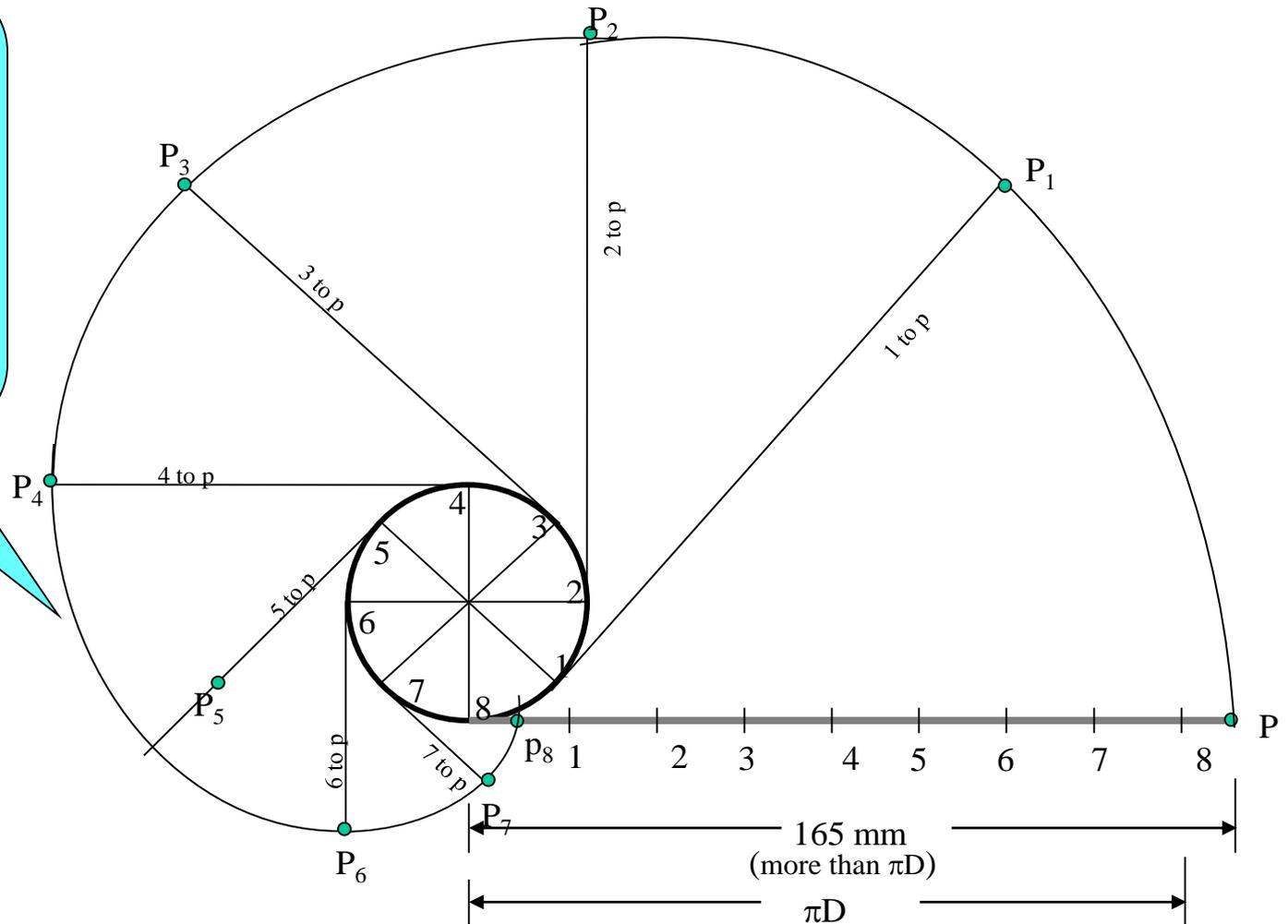
Problem 18: Draw Involute of a circle.
String length is MORE than the circumference of circle.

Solution Steps:

In this case string length is more than πD .

But remember!

Whatever may be the length of string, mark πD distance horizontal i.e. along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.



Problem 19: Draw Involute of a circle.

String length is LESS than the circumference of circle.

INVOLUTE OF A CIRCLE

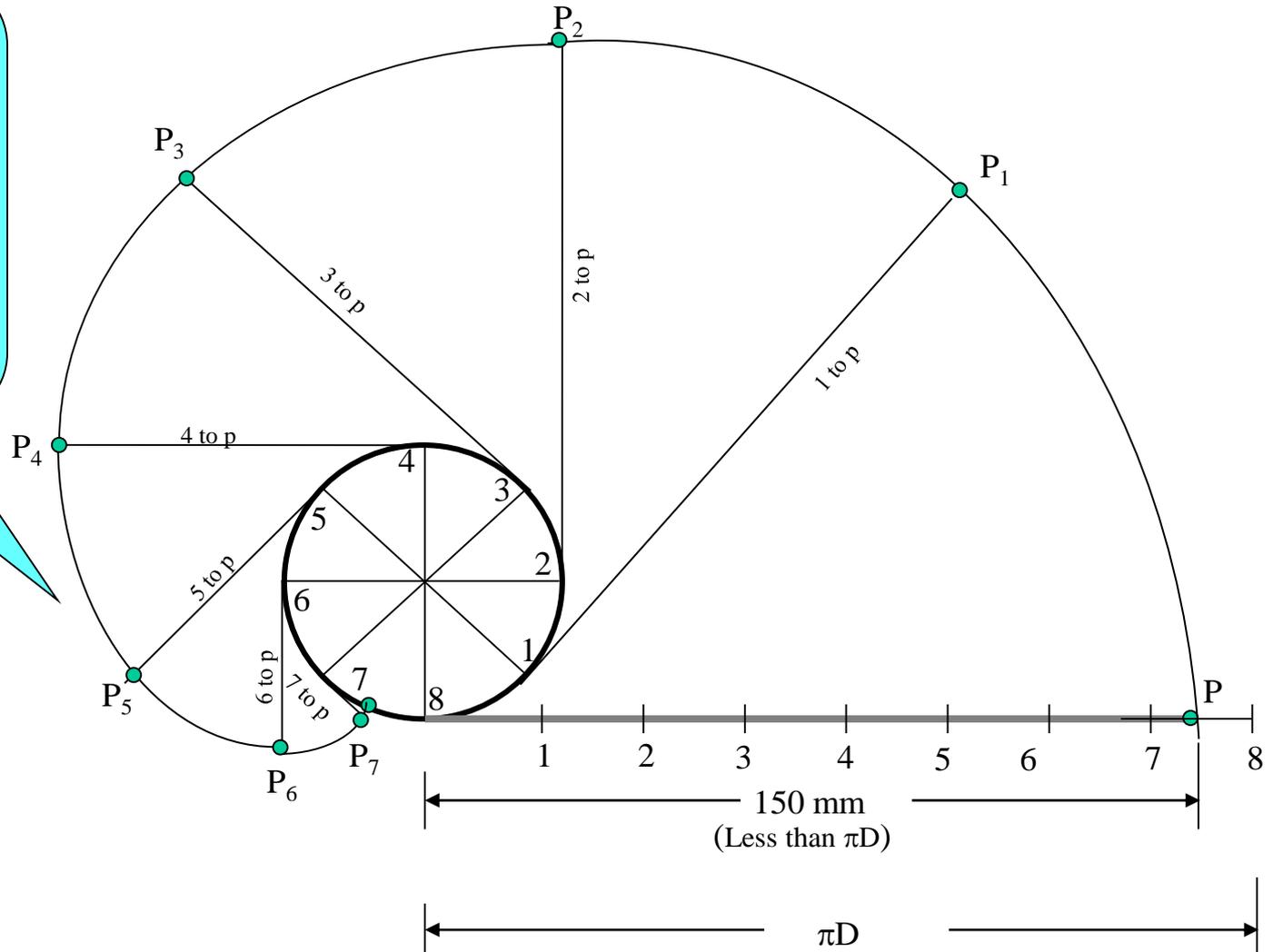
String length LESS than πD

Solution Steps:

In this case string length is Less than πD .

But remember!

Whatever may be the length of string, mark πD distance horizontal i.e. along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.

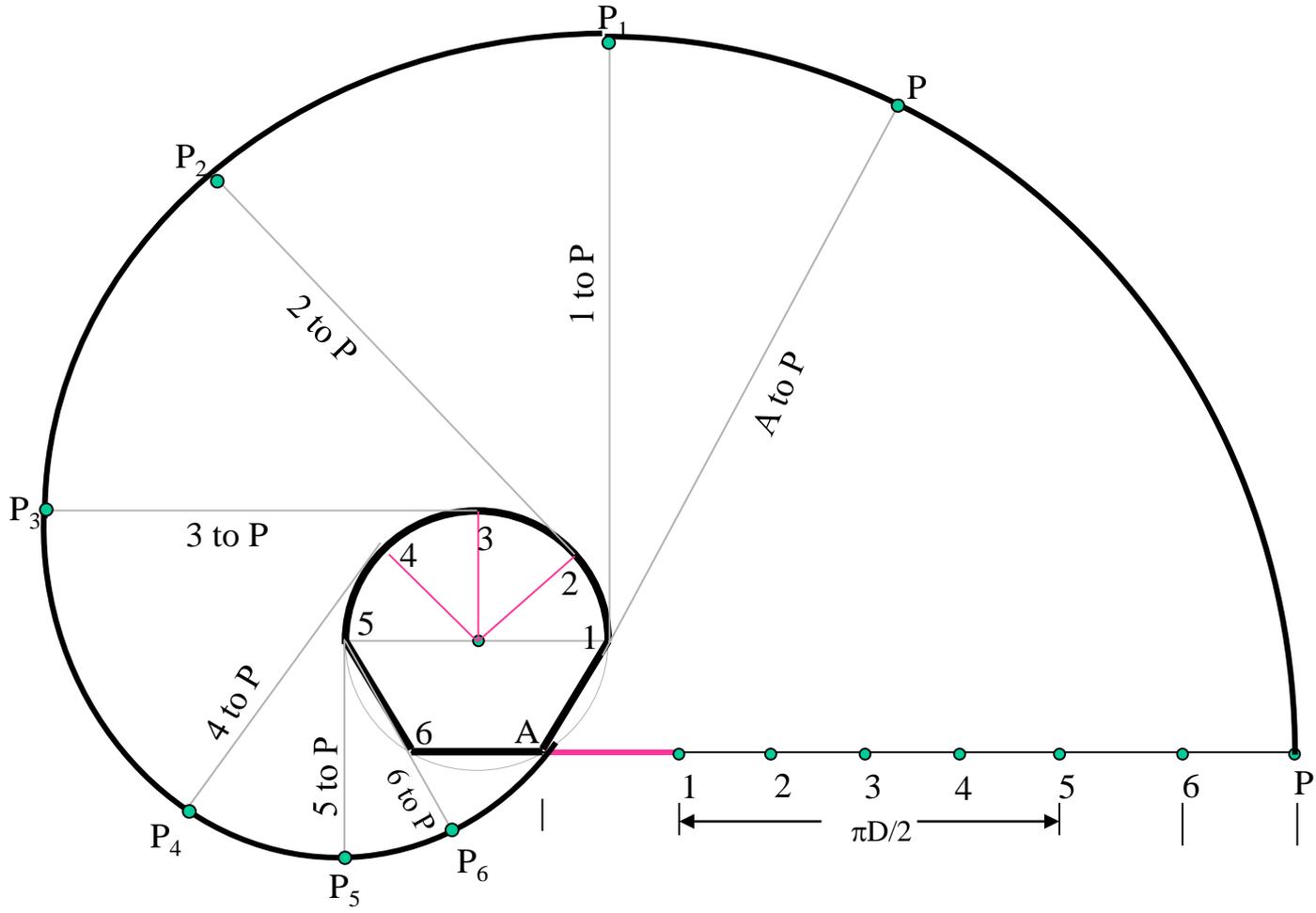


PROBLEM 20 : A POLE IS OF A SHAPE OF HALF HEXAGON AND SEMICIRCLE.
 A STRING IS TO BE WOUND HAVING LENGTH EQUAL TO THE POLE PERIMETER
 DRAW PATH OF FREE END *P* OF STRING WHEN WOUND COMPLETELY.
 (Take hex 30 mm sides and semicircle of 60 mm diameter.)

**INVOLUTE
 OF
 COMPOSIT SHAPED POLE**

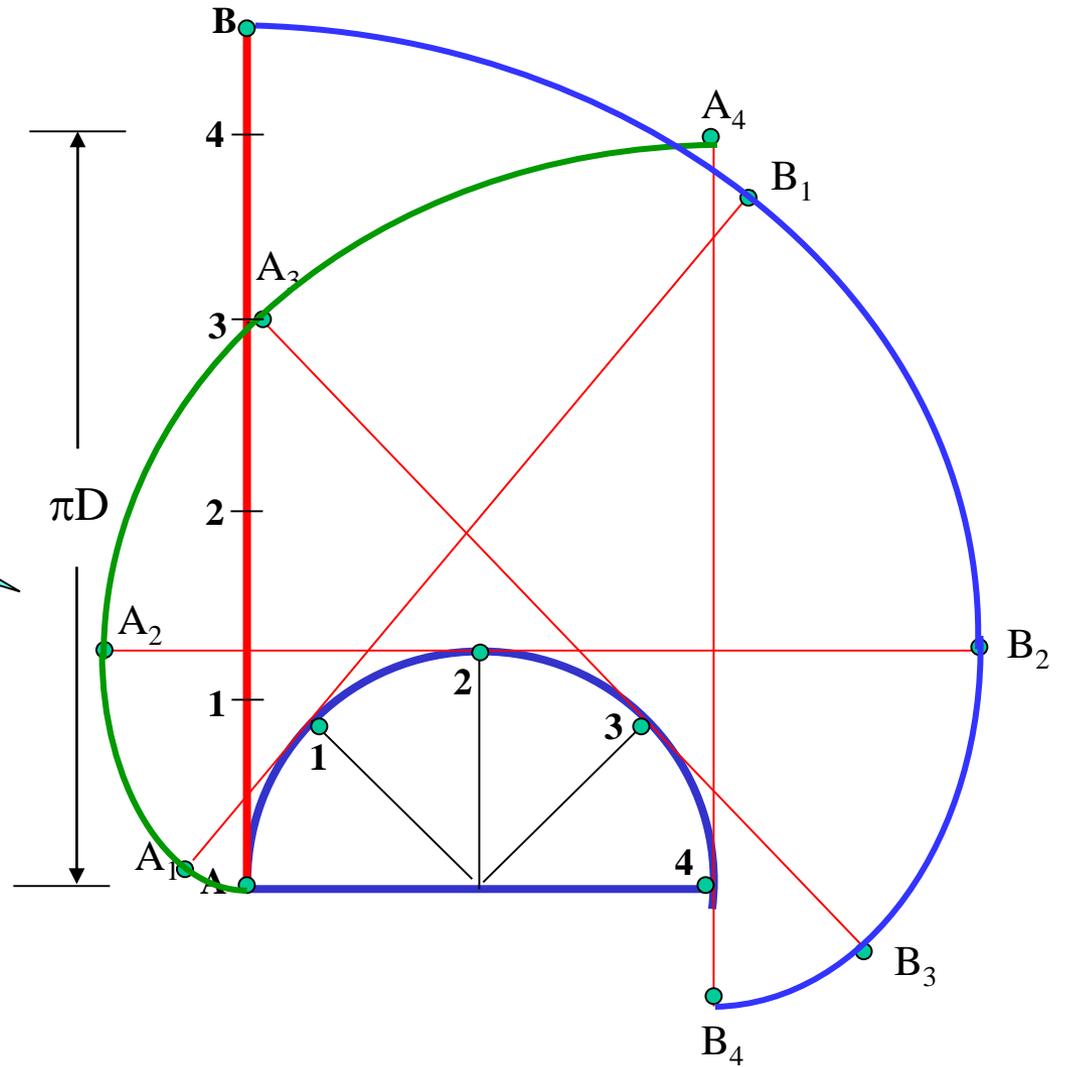
SOLUTION STEPS:

- Draw pole shape as per dimensions.
- Divide semicircle in 4 parts and name those along with corners of hexagon.
- Calculate perimeter length.
- Show it as string AP.
- On this line mark 30mm from A
- Mark and name it 1
- Mark $\pi D/2$ distance on it from 1
- And dividing it in 4 parts name 2,3,4,5.
- Mark point 6 on line 30 mm from 5
- Now draw tangents from all points of pole and proper lengths as done in all previous involute's problems and complete the curve.



PROBLEM 21 : Rod AB 85 mm long rolls over a semicircular pole without slipping from it's initially vertical position till it becomes up-side-down vertical. Draw locus of both ends A & B.

Solution Steps?
 If you have studied previous problems properly, you can surely solve this also. Simply remember that this being a rod, it will roll over the surface of pole. Means when one end is approaching, other end will move away from poll. **OBSERVE ILLUSTRATION CAREFULLY!**

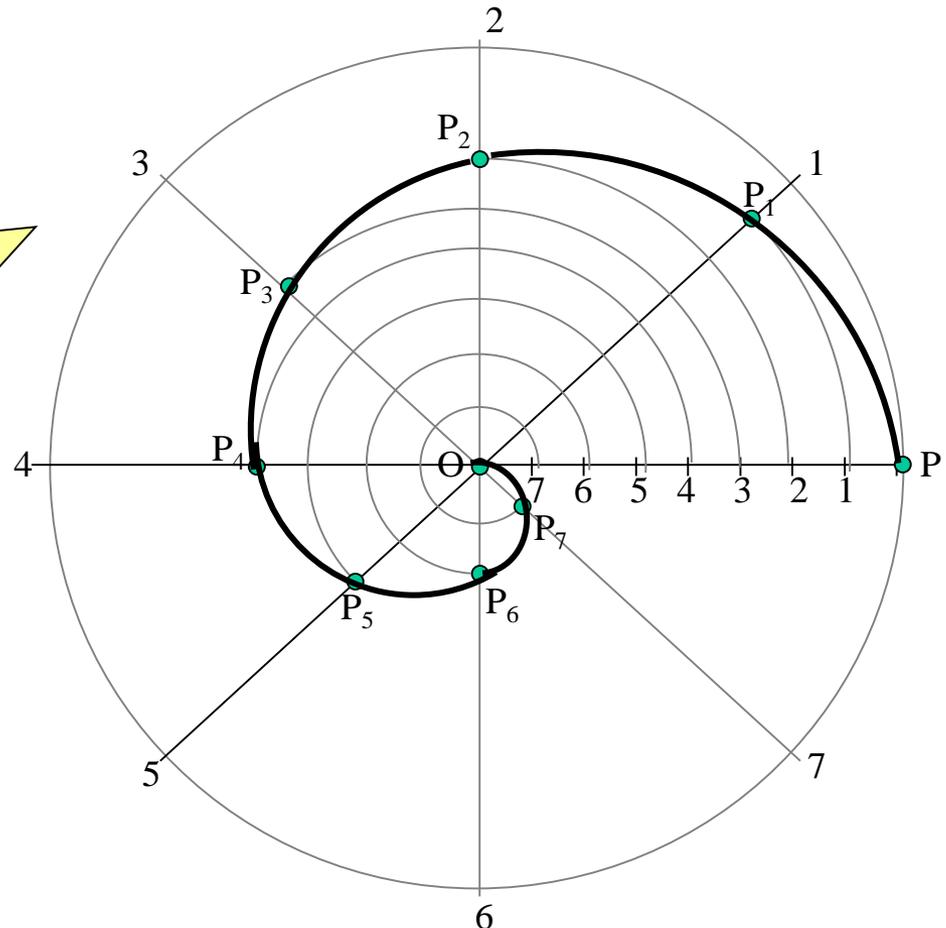


Problem 27: Draw a spiral of one convolution. Take distance PO 40 mm.

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT
AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

Solution Steps

1. With PO radius draw a circle and divide it in EIGHT parts. Name those 1,2,3,4, etc. up to 8
2. Similarly divided line PO also in EIGHT parts and name those 1,2,3,-- as shown.
3. Take o-1 distance from op line and draw an arc up to O1 radius vector. Name the point P_1
4. Similarly mark points P_2, P_3, P_4 up to P_8
 And join those in a smooth curve.
 It is a SPIRAL of one convolution.



SPIRAL of two convolutions

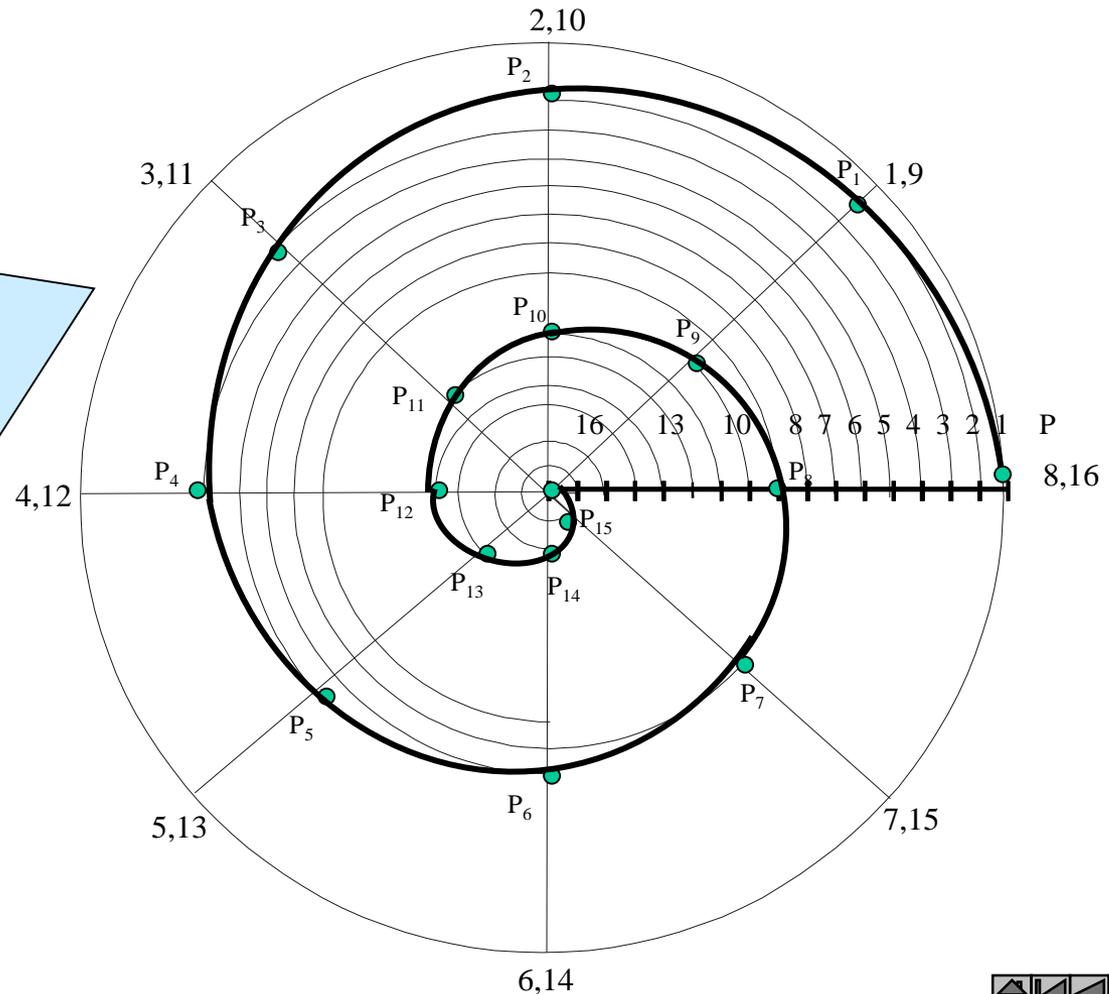
Problem 28

Point P is 80 mm from point O. It starts moving towards O and reaches it in two revolutions around it. Draw locus of point P (To draw a Spiral of TWO convolutions).

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

SOLUTION STEPS:

Total angular displacement here is two revolutions And Total Linear displacement here is distance PO.
Just divide both in same parts i.e. Circle in EIGHT parts. (means total angular displacement in SIXTEEN parts)
Divide PO also in SIXTEEN parts.
Rest steps are similar to the previous problem.

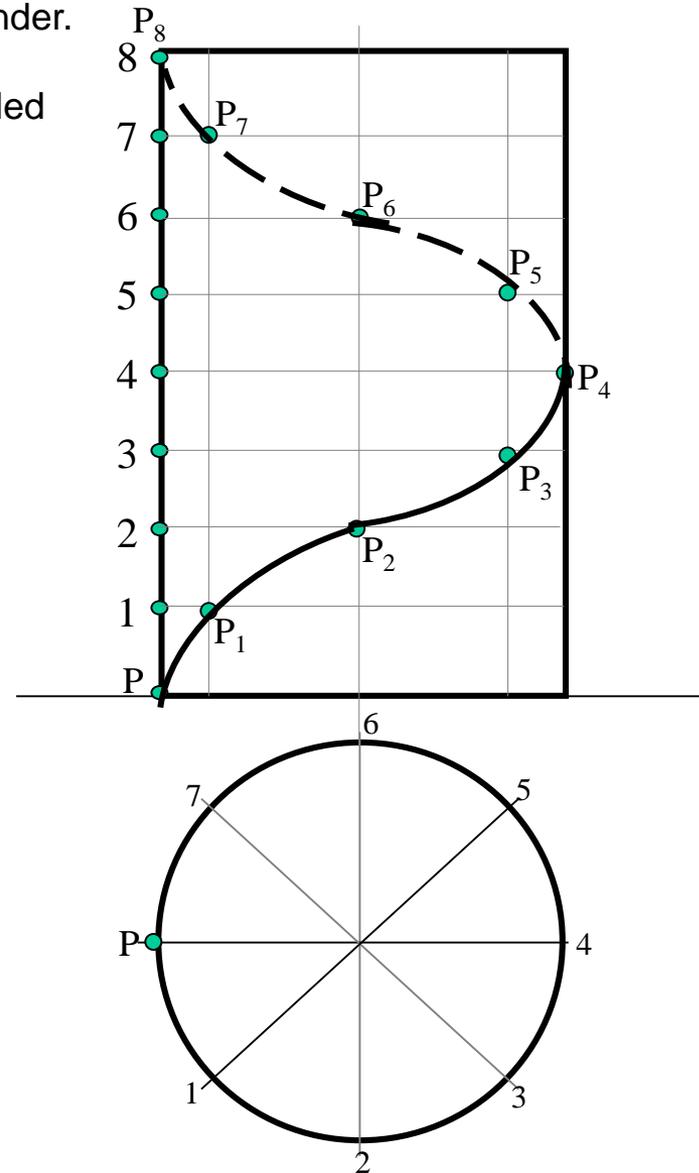


HELIX (UPON A CYLINDER)

PROBLEM: Draw a helix of one convolution, upon a cylinder.
Given 80 mm pitch and 50 mm diameter of a cylinder.
(The axial advance during one complete revolution is called
The *pitch* of the helix)

SOLUTION:

- Draw projections of a cylinder.
- Divide circle and axis in to same no. of equal parts. (8)
- Name those as shown.
- Mark initial position of point 'P'
- Mark various positions of P as shown in animation.
- Join all points by smooth possible curve.
- Make upper half dotted, as it is going behind the solid and hence will not be seen from front side.



HELIX (UPON A CONE)

PROBLEM: Draw a helix of one convolution, upon a cone, diameter of base 70 mm, axis 90 mm and 90 mm pitch.
(The axial advance during one complete revolution is called The *pitch* of the helix)

SOLUTION:

Draw projections of a cone

Divide circle and axis in to same no. of equal parts. (8)

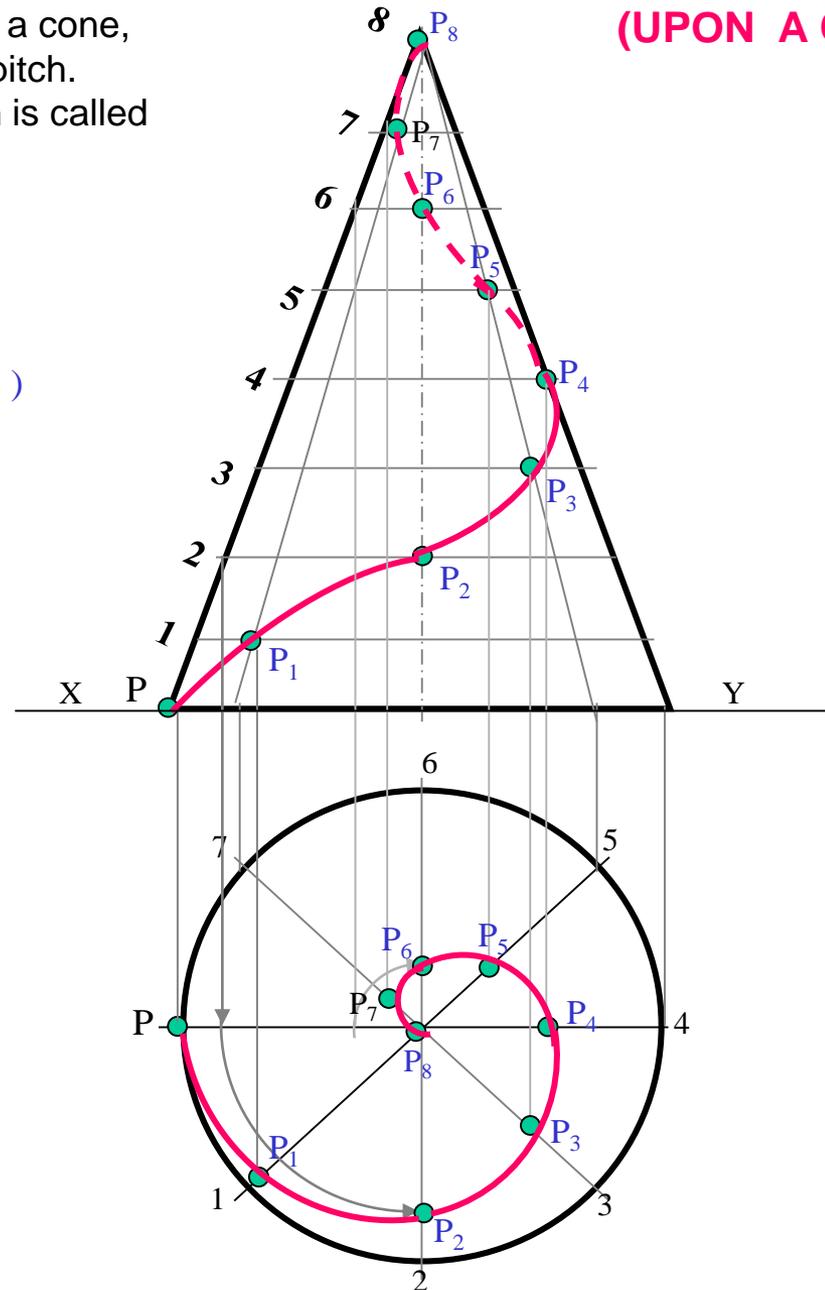
Name those as shown.

Mark initial position of point 'P'

Mark various positions of P as shown in animation.

Join all points by smooth possible curve.

Make upper half dotted, as it is going behind the solid and hence will not be seen from front side.



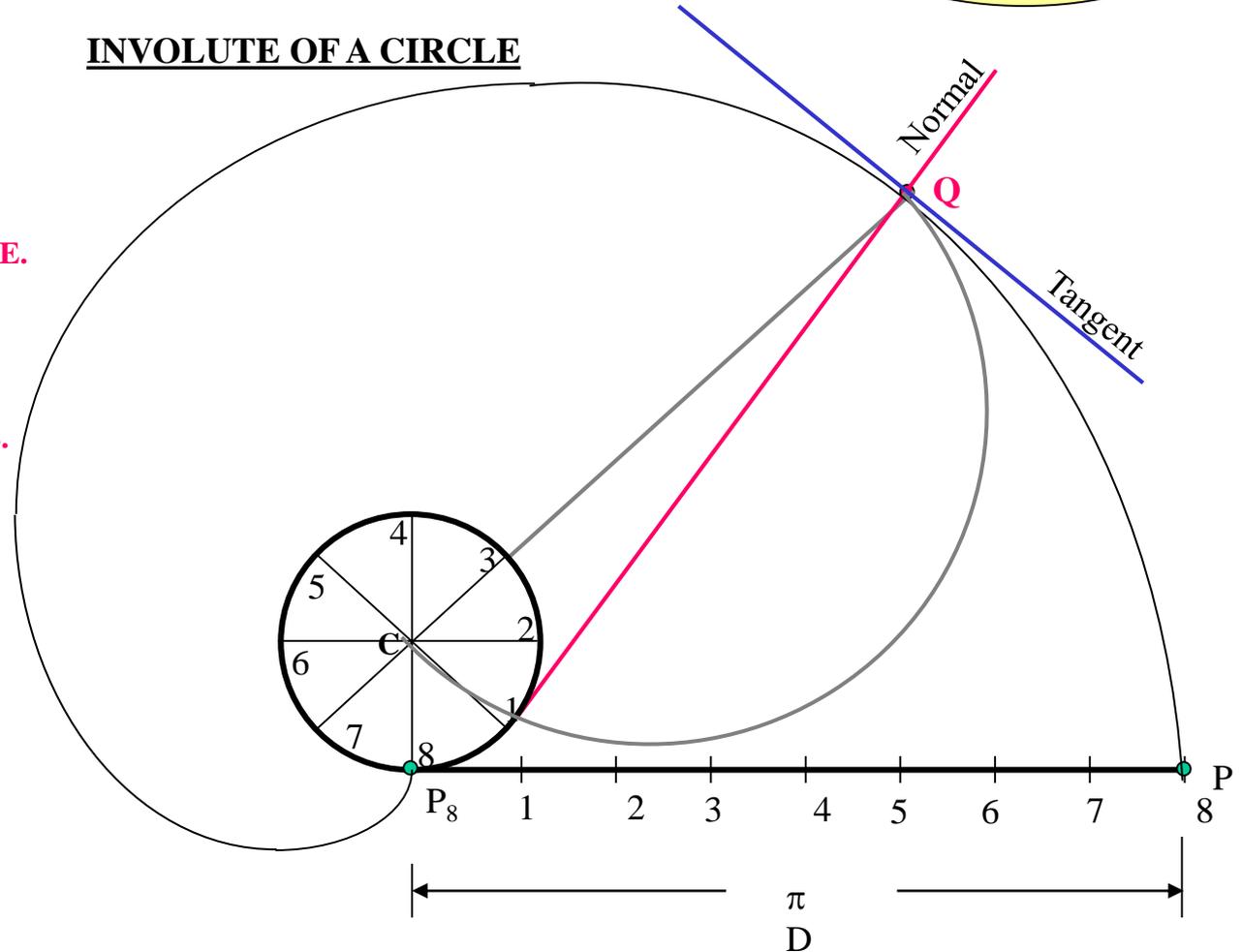


Involute Method of Drawing Tangent & Normal

STEPS:
 DRAW INVOLUTE AS USUAL.
 MARK POINT **Q** ON IT AS DIRECTED.
 JOIN **Q** TO THE CENTER OF CIRCLE **C**.
 CONSIDERING **CQ** DIAMETER, DRAW
 A SEMICIRCLE AS SHOWN.

INVOLUTE OF A CIRCLE

MARK POINT OF INTERSECTION OF
 THIS SEMICIRCLE AND POLE CIRCLE
 AND JOIN IT TO **Q**.
 THIS WILL BE **NORMAL TO INVOLUTE**.
 DRAW A LINE AT RIGHT ANGLE TO
 THIS LINE FROM **Q**.
IT WILL BE TANGENT TO INVOLUTE.





CYCLOID

Method of Drawing Tangent & Normal

STEPS:

DRAW CYCLOID AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

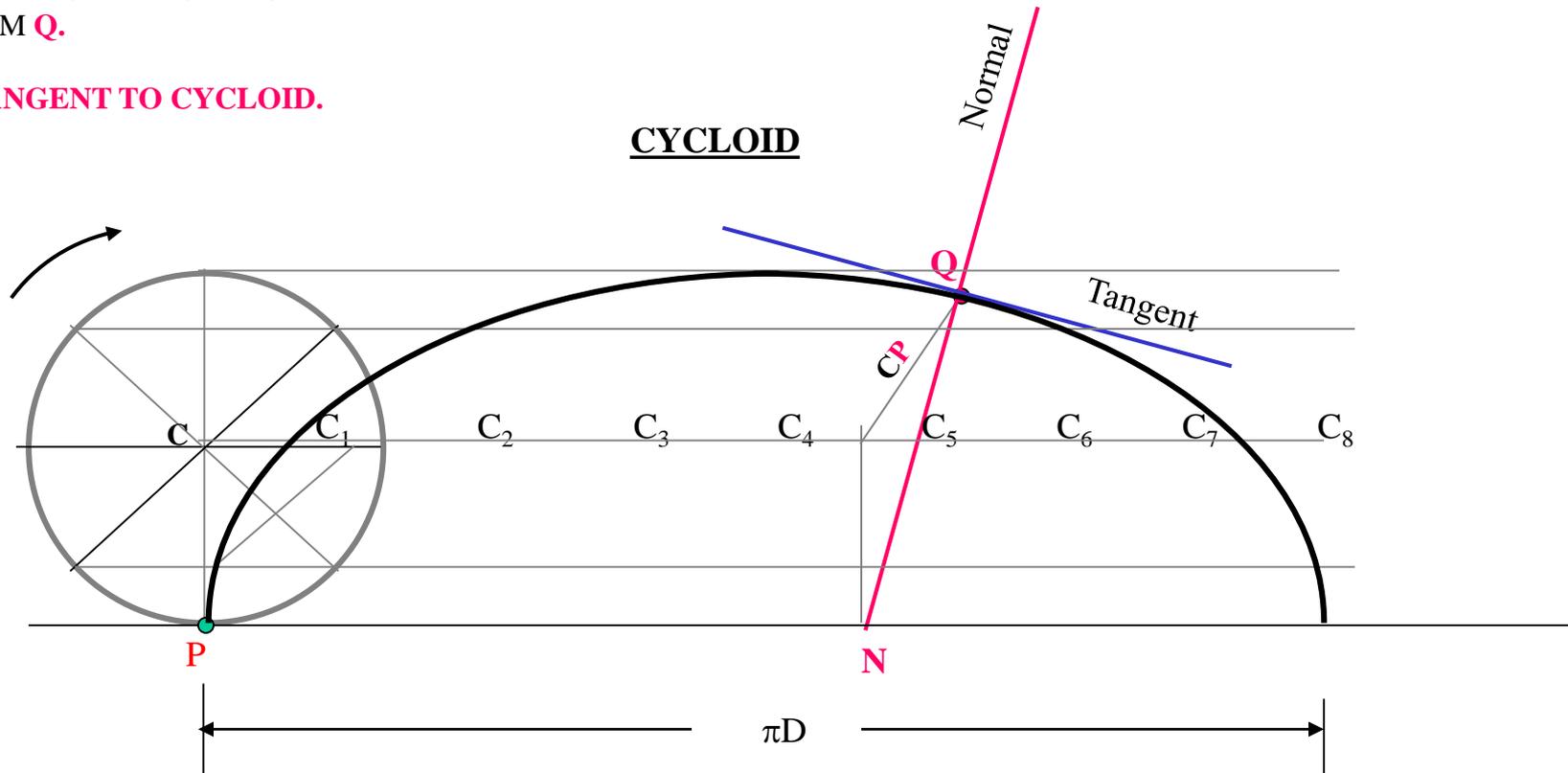
WITH CP DISTANCE, FROM **Q**. CUT THE POINT ON LOCUS OF **C** AND JOIN IT TO **Q**.

FROM THIS POINT DROP A PERPENDICULAR ON GROUND LINE AND NAME IT **N**

JOIN **N** WITH **Q**. THIS WILL BE **NORMAL TO CYCLOID**.

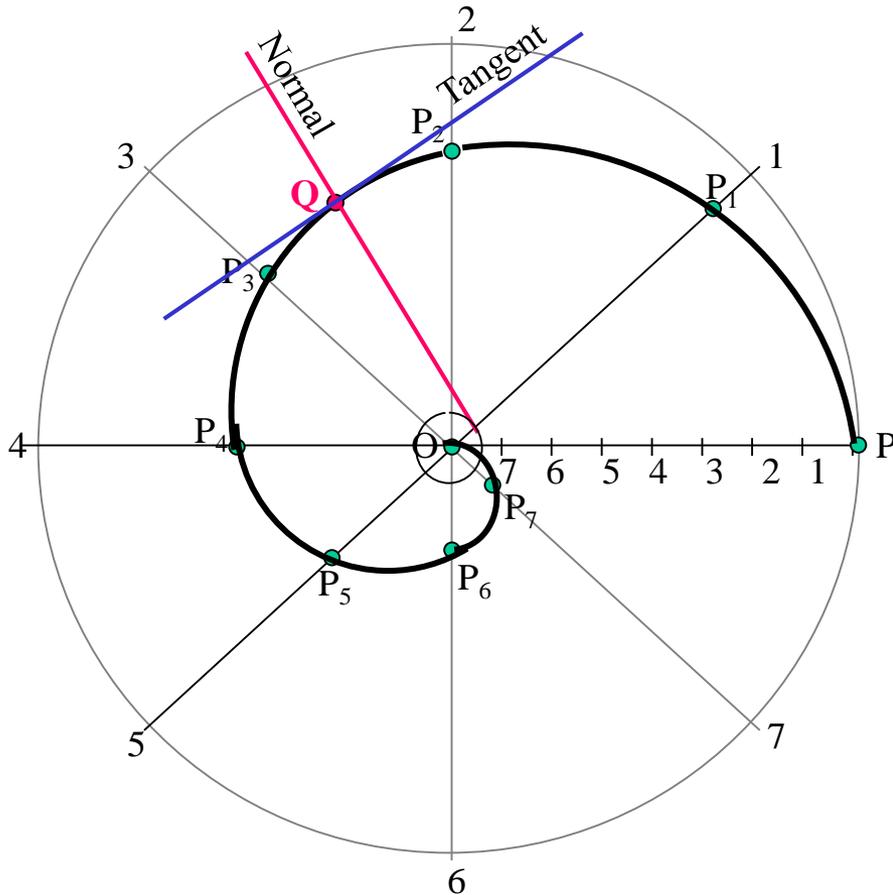
DRAW A LINE AT RIGHT ANGLE TO THIS LINE FROM **Q**.

IT WILL BE TANGENT TO CYCLOID.



Spiral. Method of Drawing Tangent & Normal

SPIRAL (ONE CONVOLUTION.)



$$\begin{aligned}
 \text{Constant of the Curve} &= \frac{\text{Difference in length of any radius vectors}}{\text{Angle between the corresponding radius vector in radian.}} \\
 &= \frac{OP - OP_2}{\pi/2} = \frac{OP - OP_2}{1.57} \\
 &= 3.185 \text{ m.m.}
 \end{aligned}$$

STEPS:

- *DRAW SPIRAL AS USUAL.
DRAW A SMALL CIRCLE OF RADIUS EQUAL TO THE CONSTANT OF CURVE CALCULATED ABOVE.
- * LOCATE POINT **Q** AS DISCRIBED IN PROBLEM AND THROUGH IT DRAW A TANGENT TO THIS SMALLER CIRCLE. THIS IS A **NORMAL** TO THE SPIRAL.
- *DRAW A LINE AT RIGHT ANGLE
- *TO THIS LINE FROM **Q**.
IT WILL BE TANGENT TO CYCLOID.